BMS INSTITUTE OF TECHNOLOGY AND MANAGEMENT

(Affiliated to the Visvesvaraya Technological University, Belagavi)

Department of Mathematics

COURSE DESIGN, DELIVERY AND ASSESMENT

Course code and title: 18MAT11 and **CALCULUS AND LINEAR ALGEBRA**	Course Credits: **(3:2:0) 4**
CIE: **50 Marks reduced to 30 Marks**	SEE: **100 Marks reduced to 60 Marks**
No. of Theory hours: 50	Lab support: NIL
Prepared by: **Mrs. Sreelakshmi T K**	Date: 07-07-2020
Reviewed by: Dr. Chethan A S	Date: 28-8-2020

Course Outcomes:

On completion of this course, students are able to:

CO1: Apply the knowledge of calculus to solve problems related to polar curves and its applications in determining the bentness of a curve.

CO2: Learn the notion of partial differentiation to calculate rates of change of multivariate functions and solve problems related to composite functions and Jacobians.

CO3: Apply the concept of change of order of integration and variables to evaluate multiple integrals and their usage in computing the area and volumes

CO4: Solve first order linear/nonlinear differential equation analytically using standard methods

CO5: Make use of matrix theory for solving system of linear equations and compute Eigen values and eigenvectors required for matrix diagonalization process.

Mapping Course Outcomes with Program Outcomes:

Course Outcomes	Program Outcomes											
	1	2	3	4	5	6	7	8	9	10	11	12
1	3	2	1									
2	3	2	1									
3	3	2	1									
4	3	2	1									
5	3	2	1									

Course Contents and Lecture Schedule:

Lesson/ Session No.	Topics	No. of Hours
1.	**(I_IA PORTIONS)** **Differential Calculus-1:** Review of elementary calculus, **polar curves:** Angle between the radius vector and tangent.	1
2.	Problems on angle between the radius vector and tangent .	1
3.	Problems on angle between two curves.	1
4.	Problems on Pedal Equation.	1
5.	Curvature and Radius of curvature - Cartesian form (without proof), problems.	1
6.	Curvature and Radius of curvature – Polar form (without proof), problems.	1
7.	Centre and circle of curvature (Formulae only)-applications of evolutes and involutes.	1
8.	Problems on evolutes and involutes.	1
9.	Problems solving	1
10.	Problems solving	1
11.	**Elementary Linear Algebra:** Rank of a matrix by elementary transformations.	1
12.	Solution of system of linear equations-consistency-problems	1
13.	Solution of a system of non- homogeneous equations by Gauss elimination method- Problems.	1
14.	Gauss - Jordan method-Problems.	1
15.	Gauss Seidel method- Problems.	1
16.	Eigen values and Eigen vectors – problems.	1
17.	Rayleigh's Power method, problems.	1
18.	Diagonalization of square matrix of order two, problems.	1
19.	Problems solving	1
20.	Problems solving (I_IA PORTIONS)	1
21.	**(II_IA PORTIONS)** **Differential Calculus-2:** Taylor's Theorem for a function of single variable, problems.	1
22.	Maclaurin's series, problems.	1
23.	Indeterminate forms, L-Hospital's Rule, problems.	1
24.	Problems on L-Hospital's Rule and Partial Derivatives.	1
25.	Problems on Total derivatives and Partial Differentiation of Composite functions.	1
26.	Maxima and minima for a function of 2 variables, problems and applications with examples.	1
27.	Method of Lagrange multipliers with one subsidiary condition, problems.	1
28.	Jacobians – problems.	1
29.	Problems solving	1
30.	Problems solving	1

31.	**Ordinary differential equations(ODE's)of first order:** Exact Differential Equations, problems.	1
32.	Equations reducible to Exact Differential Equations, problems.	1
33.	Bernoulli's differential equations, problems.	1
34.	Orthogonal trajectories, problems.	1
35.	Problems on Newton's Law of Cooling and L-R circuits. **(II_IA PORTIONS)**	1
36.	**(III_IA PORTIONS)** Nonlinear differential equations : Introduction to general and singular solutions: solvable for p.	1
37.	Clairaut's equation, problems.	1
38.	Problems on equations reducible to Clairaut's form.	1
39.	Problems solving	1
40.	Problems solving	1
41.	**Integral Calculus :** Evaluation of double integrals.	1
42.	Evaluation of triple integrals.	1
43.	Evaluation of double integrals by changing the order of integration.	1
44.	Evaluation of double integrals by changing into polar coordinates.	1
45.	Applications to find area, volume by using double integrals	1
46.	Applications to find Centre of gravity by using double integrals	1
47.	**Beta and Gamma functions:** Definition, properties & relation between them.	1
48.	Problems on Beta and Gamma functions	1
49.	Problems solving	1
50.	Problems solving(III_IA PORTIONS)	1
	Total number of Lecture hours	**50**

Course Delivery:

1. Class room teaching using board
2. Cooperative learning methods
3. Tutorial/Remedial classes

Assessment of Course Outcomes:

		What	Frequency	Max Marks	Evidence collected	Course Outcomes
Direct Assessment Methods	CIE	Internal assessment tests	Thrice (Average of all the three tests will be considered)	30	Blue books	1, 2, 3, 4 & 5
		Assignment	Twice (Average of the two will be considered)	05	Assignment reports	1, 2, 3, 4 & 5
		Quiz	Once during the course	05	Quiz answers	1, 2, 3, 4 & 5
	SEE	Standard examination covering full syllabus	Once at the End of course	100	Answer scripts	1, 2, 3, 4 & 5
Indirect Assessment Methods	Students feedback about the Delivery of the course		Twice during the course	-	Feedback forms	1, 2,3 & 5
	End of course survey (On Course contents, Quality of Delivery and Assessment methods)		Once at the End of course	-	Response through Questionnaire	1, 2, 3, 4 & 5

B M S Institute Of Technology & Management

DEPARTMENT OF MATHEMATICS

BATCH:	2019	SEM:	I
SUBJECT:	CALCULUS AND LINEAR ALGEBRA		
Faculty In-Charge:	Mrs. SREE LAKSHMI T K		

SESSION:	SEPT 2019-JAN 2020

TARGET				LEVEL
60%	STUDENTS MUST SCORE	60%	& ABOVE	3-High
55%	STUDENTS MUST SCORE	60%	& ABOVE	2-Moderate
50%	STUDENTS MUST SCORE	60%	& ABOVE	1-Low

COURSE OUTCOMES	ATTAINMENT - I A	ATTAINMENT - VTU	OVERALL ATTAINMENT
CO1	3.00	3.00	3.00
CO2	3.00	3.00	3.00
CO3	3.00	3.00	3.00
CO4	3.00	3.00	3.00
CO5	3.00	3.00	3.00

CLASS STRENGTH	60
SET TARGET	60%

			TEST-1										TEST-2										TEST-3										OBE
		CO No.	1	1	1	5	5	5	1	5			2	2	2	2	5	5	5	2			4	3	3	3	2	2	3	4			5
		Question No.	1	2	3	4	5	6	7	8	9	10	1	2	3	4	5	6	7	8	9	10	1	2	3	4	5	6	7	8	9	10	Activity / Assignment
		MAXIMUM MARKS FOR QUESTION	10	10	10	10	10	10	10	10			10	10	10	10	10	10	10	10			10	10	10	10	10	10	10	10			5
Sl No.	USN	Name																															
1		DIVYANI JAIN	10			10	10	4	10	10			8	10	10			10	10	6				10		10		10	7	10			
2		GADDAM SHAIK MOHAMMED IRSHAD	3			1	3		3					1	9				4					10			10			6			
3		GOSALIYA YASH PARESHK	10		10		8		10	7			5	6		10	5		6	10			8			10	10		9	10			
4		H P VIRAJ	8					5	8	6			1	2	4				7	7			9	6		10	10	6	7	10			
5		JARUGU GNANADEEP	10		10		10	10	10	10			10		10			6	10	10			10	10		10		10	10	10			
6		JUEL MATHAIS GEORGE	10			10		8	5	10			10		10		9		10	10				10	10		9		10	10			
7		K SWETHA	10		10		3	10	10	10			2	10	9	10		6	10	7			10	10	10	10		10	10	10			
8		KARTHIK N S	10			10	1	10	10	10			7		10			6	7	10			10		10			10	10	10			
9		KIRAN KUMAR M K		10		10	4		5	10			7	7	10			5	10	10				10	10	10		10	10	10			
10		KISHAN NUNNA	10		1	10	10		4	10				6	8			10	8	4				10	10	10		10	10	9			
11		KRISHNA C	10		10	8		10	9	10			9		9			8	10	10			10	10	10		10		10	10			
12		KUNAL GAURAV	10		4		10		8	10				8	1	1	9		8	10				10		10	10		8	10			
13		KURAKULA CHARAN TEJA		10	10		7	7	10	10			5		10			4	4	5			10	10		10	10		10	8			
14		KUSUMA P J	8	8	2	10	9	4	10	10				10	10		5	4	4	5			10	10		10	10	10	6	10			
15		LIKITH S	7			10	8			4				1	2				3				10							10			
16		LIKITHA B	10			3	7		10	6			5		3				4	5				10		10	10		6	9			
17		LOKESH BABU MEDARAM	3				8		7	8				2	1				7					10		7	10		10	9			
18		M NIHAR CHAITHANYA	6	2	4		5		4	10				8	4	1		3	4	5				10	10	10	10		10	9			
19		MAGALUR ADITHYA	8			4	8	5	8	6			7		8		5		7	2			10	1		6		10	9	10			
20		MANASA S	10		10			10	10	10				10	10			10	10	10				10		10		10	10	10			
21		MARAKA HAMSINI	10				2	8	6	10			9		10		2	6	10	10				10	10	10	10	10	10	10			
22		MEGHNA ARORA	10		6	10		8	10	7			10	10	10			10	6	7			10			10		10	10	9			
23		MONISH KUMAR N	10	10	2		10	4	8	8					2		6	2	4				10		10	10	10	6	10	9			
24		N S K K K NAGA JAYANTH	10			8		10	10	10			10		3			10	10	9			10		10	10			10	8			
25		NAVYA REDDY ALIMILI	10				6		10	3			7		10			6	10	9				10	10			10	10	9			
26		NISHA P	10		10		10	10	10	10			2	8	10	2		8	10	10			10	10	10	10	10	10	9	10			
27		POLU SATHWIK REDDY	10		10			10	10	6				10	10			10	10	10				10	10	10	10		10	10			
28		POORNESHWARI G V	10		4		9	6	5	10				10	10			6	10	5			10	10		10		10	10	6			
29		PRANEETH P BAVANNAVA	10			10		9	6	3			10					8	6	3				10		10	10		3	7			
30		PRERANA R BHAT	10		10		10	8	10	10				4	10			10	6	5				10		10		10	7	6			
31		RACHITA K A	10			10		10	10	7				8	10		10		10	9			10	10	10		10		10	10			
32		RAGHAVENDRA D	10					5	10	10			9		10			2	10	10				10		10		10	7	9			
33		RAJASHEKARA V M	10		1		4		2	3			8		3			1	8	4				10		10	10	10	7	10			
34		RAJAT MUNDHRA	10		10			10	10	10			10		10			8	10	10			10			10	10	10	10	10			
35		SIRIGIRI SRAVANI	10		10			7	5	10			5		8			10	7	7			10			10		10	10	10			
36		BAISANI SREENIJA	10		10			10	10	10				10	10		10		9	8			10		10			10	10	10			
37		MANCHA NITHISH KUMAR					6			7					4								5		10		7			3			
38		SUJATHA		10	8	2		10	5	10			1		4	10	5	2	10	10			10	10		10	10	10	10	10			
39		AYESHA ANJUM	10			10		10	10	7			6		2	2		2	5	10			10	10		10	10	10	10	10			
40		PRAJWAL B P	10	7	10				8	10			7		5	5	7		5	10				10	10	10	10	10	5	10			
41		BHAVYA SHREE N	10			10	10		5	10			3	1	10			10	10	10			10	10		10		10	4	10			
42		C MANOJ	10			10		6	10	10			5		1			1	10	7			10		10		10		6	10			
43		MADHUSUDHAN M S	1				6			6									6				3	10			7			10			
44		IMPANA BHARADWAJ	10		2		8		6	4				10	3			6	10	6				10		5	10		7	10			
45		MANJUNATH L	6	8		10		10	6	5			7	5	7			4	10	9			10	10	10		10	1	10	10			
46		GOWTHAM C	10		10		8	10	7	10			9		8			1	6	2			10			10	10		7	10			
47		S NIRUPPAMA SHREE	10			10		10	10	10				8	8		10		8	10			10		10	10		10	10	10			
48		HARISH K M		3	7				10	5				5			5	5					10		10	10	4		10	10			
49		PRABHAT I PATIL	10		10			8	10	10			7		6		10		7	7			10		10		10		10	10			
50		CHARITRA		7		10		10	7	4			6		10			6	8	7			10	10	6			10	10	9			
51		MOHAMMED FAHAD R	10		10			10	10	10			8		4	2	8		9	10			10	10	10	10	10		10	9			
52		AMARESHANGADI	10				3		2				6		10			6	10	9				10		1	10			10			
53		RAHUL S R	10		10		7		10	4			10		10			7	7	6				10	10			10	10	10			
54		GANTASHALA VARUN		10			7		5	5			5	1	1				5	4				10		10	10		5	3			
55		PAVITRA RAMAKRISHNAN	10			10		10	6	10			7		8		7		10	5				10		10		10	10	9			
56		MANOJ KUMAR M	10	5	9		4	5	10	10			6	4	10		3	4	7	8			9	7		10	10	10	10	9			
57		ARYAN GUPTA	10					8	7	4			7		10				3	3				10	10			6	7	10			
58		K MANISH KUMAR PATRO	5					9		6			6		10				10	2				10		10	10		6	6			
59		SIDDHAM KUMAR TIWAR	4			10		8		3				1	1				8					10				7	5	9			
60		SAGAR KUMAR SINGH	5				9			1						2		6		2				10	4			10	10	9			

Sl No.	CO1 TOTAL	CO1 PERCENT %	CO1 Target >=60%	CO2 TOTAL	CO2 PERCENT %	CO2 Target >=60%	CO3 TOTAL	CO3 PERCENT %	CO3 Target >=60%	CO4 TOTAL	CO4 PERCENT %	CO4 Target >=60%	CO5 TOTAL	CO5 PERCENT %	CO5 Target >=60%	ALL COs THEORY EXT Grade Point	100
1	20	100.00%	Y	44	88.00%	Y	27	90.00%	Y	10	100.00%	Y	54	90.00%	Y	60	100
2	6	30.00%		20	66.67%	Y	10	100.00%	Y	6	60.00%	Y	8	26.67%		15	25
3	30	100.00%	Y	41	82.00%	Y	19	95.00%	Y	18	90.00%	Y	26	65.00%	Y	58	97
4	16	80.00%	Y	30	50.00%		23	76.67%	Y	19	95.00%	Y	18	60.00%	Y	43	72
5	30	100.00%	Y	40	100.00%	Y	30	100.00%	Y	20	100.00%	Y	46	92.00%	Y	60	100
6	15	75.00%	Y	39	97.50%	Y	30	100.00%	Y	10	100.00%	Y	47	94.00%	Y	51	85
7	30	100.00%	Y	48	80.00%	Y	40	100.00%	Y	20	100.00%	Y	39	78.00%	Y	46	77
8	20	100.00%	Y	37	92.50%	Y	20	100.00%	Y	20	100.00%	Y	44	73.33%	Y	58	97
9	15	75.00%	Y	44	88.00%	Y	40	100.00%	Y	10	100.00%	Y	39	78.00%	Y	43	72
10	15	50.00%		28	70.00%	Y	40	100.00%	Y	9	90.00%	Y	48	96.00%	Y	52	87
11	29	96.67%	Y	38	95.00%	Y	30	100.00%	Y	20	100.00%	Y	46	92.00%	Y	56	93
12	22	73.33%	Y	30	60.00%	Y	28	93.33%	Y	10	100.00%	Y	37	92.50%	Y	55	92
13	30	100.00%	Y	30	75.00%	Y	30	100.00%	Y	18	90.00%	Y	32	64.00%	Y	55	92
14	28	70.00%	Y	45	90.00%	Y	26	86.67%	Y	20	100.00%	Y	46	65.71%	Y	54	90
15	7	70.00%	Y	3	15.00%		0	0.00%		20	100.00%	Y	25	62.50%	Y	39	65
16	20	100.00%	Y	23	57.50%		26	86.67%	Y	9	90.00%	Y	20	50.00%		48	80
17	10	50.00%		13	43.33%		27	90.00%	Y	9	90.00%	Y	23	76.67%	Y	23	38
18	16	40.00%		28	56.00%		40	100.00%	Y	9	90.00%	Y	22	55.00%		49	82
19	16	80.00%	Y	27	67.50%	Y	16	53.33%		20	100.00%	Y	35	58.33%		38	63
20	30	100.00%	Y	40	100.00%	Y	30	100.00%	Y	10	100.00%	Y	40	100.00%	Y	56	93
21	16	80.00%	Y	49	98.00%	Y	40	100.00%	Y	10	100.00%	Y	38	63.33%	Y	48	80
22	26	86.67%	Y	47	94.00%	Y	20	100.00%	Y	19	95.00%	Y	41	82.00%	Y	53	88
23	30	75.00%	Y	18	60.00%	Y	30	100.00%	Y	19	95.00%	Y	34	56.67%		38	63
24	20	100.00%	Y	32	80.00%	Y	20	100.00%	Y	18	90.00%	Y	48	96.00%	Y	58	97
25	20	100.00%	Y	36	90.00%	Y	30	100.00%	Y	9	90.00%	Y	25	62.50%	Y	59	98
26	30	100.00%	Y	52	74.29%	Y	39	97.50%	Y	20	100.00%	Y	48	96.00%	Y	54	90
27	30	100.00%	Y	40	100.00%	Y	40	100.00%	Y	10	100.00%	Y	36	90.00%	Y	53	88
28	19	63.33%	Y	35	87.50%	Y	30	100.00%	Y	6	60.00%	Y	41	82.00%	Y	53	88
29	16	80.00%	Y	33	82.50%	Y	23	76.67%	Y	7	70.00%	Y	36	72.00%	Y	57	95
30	30	100.00%	Y	29	72.50%	Y	27	90.00%	Y	6	60.00%	Y	44	88.00%	Y	54	90
31	20	100.00%	Y	37	92.50%	Y	30	100.00%	Y	20	100.00%	Y	47	94.00%	Y	51	85
32	20	100.00%	Y	39	97.50%	Y	27	90.00%	Y	9	90.00%	Y	27	67.50%	Y	47	78
33	13	43.33%		35	70.00%	Y	27	90.00%	Y	10	100.00%	Y	16	40.00%		48	80
34	30	100.00%	Y	50	100.00%	Y	20	100.00%	Y	20	100.00%	Y	38	95.00%	Y	57	95
35	25	83.33%	Y	30	75.00%	Y	20	100.00%	Y	20	100.00%	Y	34	85.00%	Y	54	90
36	30	100.00%	Y	38	95.00%	Y	20	100.00%	Y	20	100.00%	Y	39	97.50%	Y	59	98
37	0	0.00%		11	55.00%		10	100.00%	Y	8	40.00%		13	65.00%	Y	6	10
38	23	76.67%	Y	45	75.00%	Y	30	100.00%	Y	20	100.00%	Y	39	65.00%	Y	57	95
39	20	100.00%	Y	40	66.67%	Y	30	100.00%	Y	20	100.00%	Y	34	68.00%	Y	44	73
40	35	87.50%	Y	47	78.33%	Y	35	87.50%	Y	10	100.00%	Y	22	73.33%	Y	37	62
41	15	75.00%	Y	34	68.00%	Y	24	80.00%	Y	20	100.00%	Y	50	100.00%	Y	46	77
42	20	100.00%	Y	23	57.50%		16	80.00%	Y	20	100.00%	Y	37	74.00%	Y	38	35
43	1	10.00%		7	70.00%	Y	10	100.00%	Y	13	65.00%	Y	18	60.00%	Y	21	70
44	18	60.00%	Y	29	72.50%	Y	22	73.33%	Y	10	100.00%	Y	28	70.00%	Y	42	85
45	20	66.67%	Y	39	65.00%	Y	30	100.00%	Y	20	100.00%	Y	39	78.00%	Y	51	95
46	27	90.00%	Y	29	72.50%	Y	17	85.00%	Y	20	100.00%	Y	35	70.00%	Y	57	95
47	20	100.00%	Y	36	90.00%	Y	30	100.00%	Y	10	100.00%	Y	48	96.00%	Y	57	95
48	20	66.67%	Y	9	45.00%		30	100.00%	Y	20	100.00%	Y	15	50.00%		12	20
49	30	100.00%	Y	30	75.00%	Y	20	100.00%	Y	20	100.00%	Y	35	87.50%	Y	55	92
50	14	70.00%	Y	33	82.50%	Y	26	86.67%	Y	19	95.00%	Y	38	76.00%	Y	56	93
51	30	100.00%	Y	34	68.00%	Y	40	100.00%	Y	19	95.00%	Y	37	92.50%	Y	56	93
52	12	60.00%	Y	35	87.50%	Y	11	55.00%		10	100.00%	Y	19	63.33%	Y	36	60
53	30	100.00%	Y	36	90.00%	Y	30	100.00%	Y	10	100.00%	Y	25	62.50%	Y	56	93
54	15	75.00%	Y	21	42.00%		25	83.33%	Y	3	30.00%		17	56.67%		31	52
55	16	80.00%	Y	30	75.00%	Y	30	100.00%	Y	9	90.00%	Y	47	94.00%	Y	55	92
56	34	85.00%	Y	48	80.00%	Y	27	90.00%	Y	18	90.00%	Y	33	55.00%		56	93
57	17	85.00%	Y	26	65.00%	Y	27	90.00%	Y	10	100.00%	Y	15	50.00%		44	73
58	5	50.00%		28	70.00%	Y	26	86.67%	Y	6	60.00%	Y	25	83.33%	Y	44	73
59	4	40.00%		9	30.00%		15	75.00%	Y	9	90.00%	Y	29	72.50%	Y	21	35
60	5	50.00%		14	46.67%		24	80.00%	Y	9	90.00%	Y	16	53.33%		50	83

Target >=60%	50	Target >=60%	49	Target >=60%	57	Target >=60%	58	Target >=60%	49	4748
CO1	84.75%	CO2	81.67%	CO3	96.61%	CO4	96.67%	CO5	81.67%	Avg. 79.1388888
										88.33%

BATCH	2019
SEM	I
SUB	CALCULUS AND LINEAR ALGEBRA
SESSION	SEPT 2019-JAN 2020
Class Strength	60
Set Target I A	60%
Faculty	Mrs. SREE LAKSHMI T K
Target >=60%	
3	60%
2	55%
1	50%
No. of COs	5
Set Target Uni Theo	60.00%
Set Target Uni lab	60.00%

CO RESULT	PO1	PO2	PO3	PO4	PO5	PO6	PO7	PO8	PO9	PO10	PO11	PO12	PSO1	PSO2	Delivery Mechanism

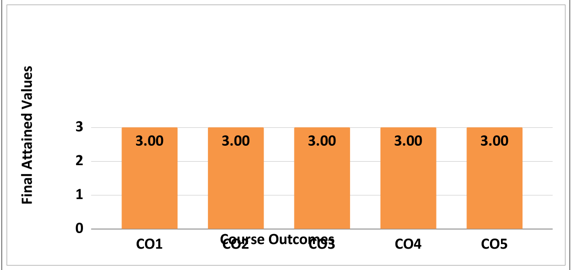
FINAL CO ATTAINMENT ANALYSIS - 18MAT11 - 2019-20 - 2019 Batch

CO1:Apply the knowledge of calculus to solve problems related to polar curves and its applications in determining the bentness of a curve.	3	3	2	1												Lecture, Tutorial
CO2:Learn the notion of partial differentiation to calculate rates of change of multivariate functions and solve related to composite functions and Jacobians.	3	3	2	1												Lecture, Tutorial
CO3: Apply the concept of change of order of integration and variables to evaluate multiple integrals and their usage in computing the area and volumes.	3	3	2	1												Lecture, Tutorial
CO4: Solve first order linear/nonlinear differential equation analytically usingstandard methods.	3	3	2	1												Lecture, Tutorial
CO5: Use concepts of matrices which is essential for several computational techniques.	3	3	2	1												Lecture, Tutorial
SUM		15	10	5	0	0	0	0	0	0	0	0	0	0	0	
Ciii*		3	2	1	0	0	0	0	0	0	0	0	0	0	0	

5	5	5	0	0	0	0	0	0	0	0	0	0	0

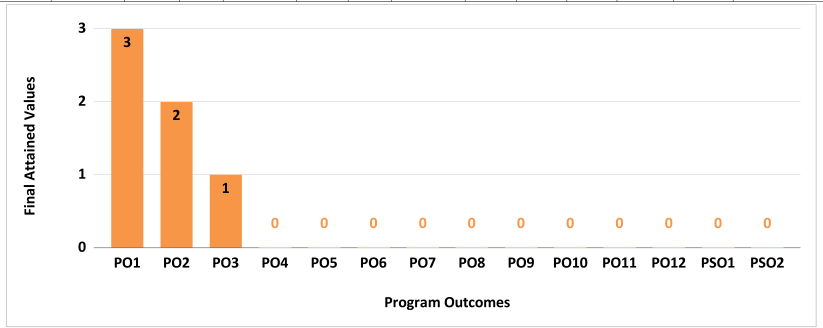
	CO RESULT	PO1	PO2	PO3	PO4	PO5	PO6	PO7	PO8	PO9	PO10	PO11	PO12	PSO1	PSO2
CO1:Apply the knowledge of calculus to solve problems related to polar curves and its applications in determining the bentness of a curve.	3.00	3	2	1	0	0	0	0	0	0	0	0	0	0	0
CO2:Learn the notion of partial differentiation to calculate rates of change of multivariate functions and solve related to composite functions and Jacobians.	3.00	3	2	1	0	0	0	0	0	0	0	0	0	0	0
CO3: Apply the concept of change of order of integration and variables to evaluate multiple integrals and their usage in computing the area and volumes.	3.00	3	2	1	0	0	0	0	0	0	0	0	0	0	0
CO4: Solve first order linear/nonlinear differential equation analytically usingstandard methods.	3.00	3	2	1	0	0	0	0	0	0	0	0	0	0	0
CO5: Use concepts of matrices which is essential for several computational techniques.	3.00	3	2	1	0	0	0	0	0	0	0	0	0	0	0
SUM		15	10	5	0	0	0	0	0	0	0	0	0	0	0
DIRECT ATTAINMENT		3	2	1	0	0	0	0	0	0	0	0	0	0	0

CO1	CO2	CO3	CO4	CO5
3.00	3.00	3.00	3.00	3.00



FINAL PO ATTAINMENT ANALYSIS - 18MAT21 - 2018-19 - 2018 Batch

PO1	PO2	PO3	PO4	PO5	PO6	PO7	PO8	PO9	PO10	PO11	PO12	PSO1	PSO2
3	2	1	0	0	0	0	0	0	0	0	0	0	0



	PO1	PO2	PO3	PO4	PO5	PO6	PO7	PO8	PO9	PO10	PO11	PO12	PSO1	PSO2
FINAL ATTAINMENT	3	2	1	0	0	0	0	0	0	0	0	0	0	0

BMS INSTITUTE OF TECHNOLOGY AND MANAGEMENT
Yelahanka, Bengaluru – 560064.
Department of Mathematics

REPORT ON TUTORIAL CLASS OF I ISE (D) HELD ON 19TH FEBRAURY, 2021

Online Tutorial class was conducted for I ISE (D) from 9:30 a.m. to 10:30 a.m. by Mrs. Sreelakshmi T.K.
Number of Students: 68
The Cooperative learning method which was introduced was as follows:

1. **BRAINSTROMING**
    - Ask students (**Individually** or in pairs or groups) to brainstorm about any concept / issue/problem
    - Give time to brainstorm (2-3 minutes).
    - Randomly call any student to share their thoughts on the issue.
    - Don't judge ideas-Purpose of brainstorming is just to get students to think about an issue


TOPIC OF DISSCUSSION: **Problems on Reducible** to exact equations
IMPACT ANALYSIS :

- Highly motivating.
- Increase task focus.
- Promotes spontaneity and creativity.
- Efficient and procedure.
- It is an intellectual activity.
- Maximum or all students can participate.
- Each student gives their personal view/ideas.
- Each idea is neither right nor wrong.
- It involves divergent thinking.

Course Coordinator
Sreelakshmi T K

HoD
Maths

ASSIGNMENT

ENGINEERING MATHEMATICS

"CALCULUS AND LINEAR ALGEBRA"

SOLUTIONS OF INTERNAL ASSESSMENT-1 PAPER.

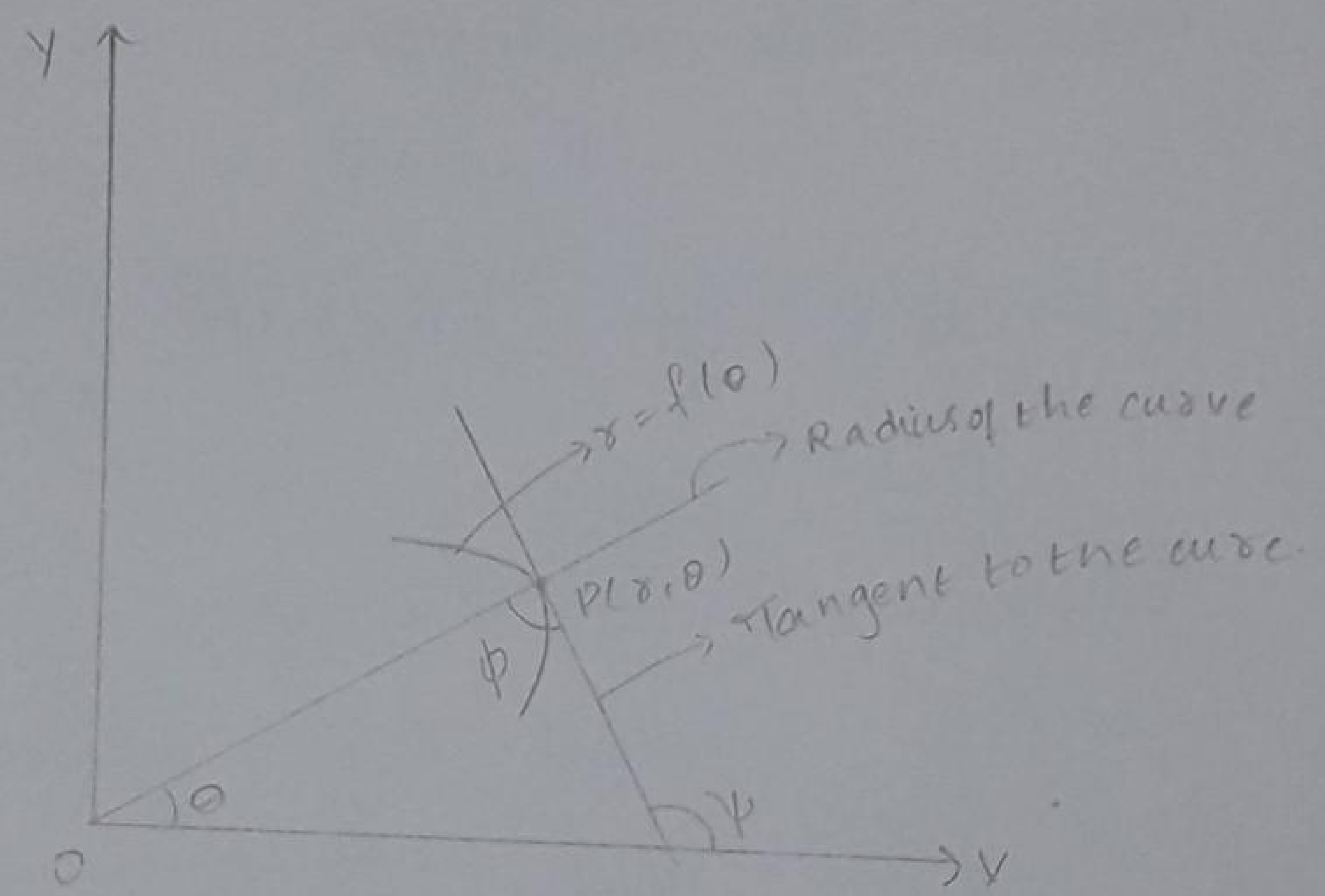
NAME: AHAANA SINGH
BATCH: ISE-D
SERIAL NO: 56.

PART-A

Q1. (i) With usual notations, prove that $\tan\phi = r\dfrac{d\theta}{dr}$

(ii) Find the angle between the curves $r=\sin\theta+\cos\theta$ and $r=2\sin\theta$

Sol.



Let there be a curve $r=f(\theta)$ and a point on it as $P(r,\theta)$

Let 'θ' be the angle between the radius vector and the tangent. and the initial line.

The relation between the cartesian and the polar coordinates is given by
$$x = r\cos\theta$$
$$y = r\sin\theta$$

$\therefore \frac{dx}{d\theta} = \cos\theta \frac{dr}{d\theta} - r\sin\theta \qquad \therefore \frac{dy}{d\theta} = \sin\theta \frac{dr}{d\theta} + r\cos\theta.$

$$\therefore \frac{dy}{dx} = \frac{\sin\theta \frac{dr}{d\theta} + r\cos\theta}{\cos\theta \frac{dr}{d\theta} - r\sin\theta}$$

On dividing numerator and denominator by $\frac{dr}{d\theta}\cos\theta$

$$\therefore \frac{dy}{dx} = \frac{\tan\theta + r\frac{d\theta}{dr}}{1 - r\frac{d\theta}{dr}\cdot\tan\theta} \quad \text{—— (1)}$$

Also, we know that, exterior angle is the sum of the opposite interior angles.

$$\Rightarrow \psi = \theta + \phi$$

But $\tan\psi = \frac{dy}{dx} = \tan(\theta+\phi) = \frac{\tan\theta + \tan\phi}{1 - \tan\theta\tan\phi}$ —— (11)

Comparing (1) and (11), we get

$\tan\phi = r\frac{d\theta}{dr}$ which can also be written as

$$\cot\phi = \frac{1}{r}\cdot\frac{dr}{d\theta}$$

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(ii) $r = \sin\theta + \cos\theta$ ⟶ curve ①

$\log r = \log(\sin\theta + \cos\theta)$

on differentiating,

$$\frac{1}{r}\cdot\frac{dr}{d\theta} = \frac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta}$$

$$\Rightarrow \cot\phi_1 = \frac{1-\tan\theta}{1+\tan\theta}$$ [on division of RHS by $\cos\theta$] [Also, $\frac{1}{r}\frac{dr}{d\theta} = \cot\phi_1$]

$$\Rightarrow \cot\phi_1 = \tan(\pi/4 - \theta)$$

$$\Rightarrow \cot\phi_1 = \cot(\pi/2 - \pi/4 + \theta)$$

$$\Rightarrow \cot\phi_1 = \cot(\pi/4 + \theta)$$

$$\Rightarrow \phi_1 = \pi/4 + \theta \quad \text{——} ①$$

$r = 2\sin\theta$

$\log r = \log 2\sin\theta = \log 2 + \log\sin\theta$

$$\Rightarrow \frac{1}{r}\cdot\frac{dr}{d\theta} = 0 + \frac{\cos\theta}{\sin\theta}$$

$$\Rightarrow \cot\phi_2 = \cot\theta.$$

$$\Rightarrow \phi_2 = \theta \quad \text{——} ⑪$$

$\therefore$ Angle between the curves $= |\phi_1 - \phi_2| = |\pi/4 + \theta - \theta| = \pi/4$ //

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Q2. (i) Show that the curves $r = a(1-\sin\theta)$ and $r = a(1+\sin\theta)$ are orthogonal.

(ii) Find the pedal equation of the curve $r^m = a^m(\sin m\theta + \cos m\theta)$

Solⁿ (i) $r = a(1-\cancel{\cos}\sin\theta)$ ⟶ curve 1

$\log r = \log a + \log(1-\sin\theta)$

$\Rightarrow \frac{1}{r}\cdot\frac{dr}{d\theta} = 0 - \frac{\cancel{\sin\theta}\cos\theta}{1-\sin\theta}$

$\Rightarrow \cot\phi_1 = \frac{-(\cos^2\theta/2 - \sin^2\theta/2)}{\cos^2\theta/2 + \sin^2\theta/2 - 2\sin\theta/2\cos\theta/2}$

$\Rightarrow \cot\phi_1 = \frac{-(\cos\theta/2 - \sin\theta/2)(\cos\theta/2 + \sin\theta/2)}{(\sin\theta/2 - \cos\theta/2)^2}$

$\Rightarrow \cot\phi_1 = \frac{\cos\theta/2 + \sin\theta/2}{\sin\theta/2 - \cos\theta/2}$

$\Rightarrow \cot\phi_1 = \frac{1+\tan\theta/2}{1-\tan\theta/2}$ (on ÷ by $\cos\theta/2$ in num & denom)

$\Rightarrow \cot\phi_1 = \tan(\pi/4 + \theta/2)$

$\Rightarrow \cot\phi_1 = \cot(\pi/2 - \pi/4 - \theta/2) = \cot(\pi/4 - \theta/2) \Rightarrow \phi_1 = \pi/4 - \theta/2$ — 1

Now, $r = a(1+\sin\theta)$

$\Rightarrow \log r = \log a + \log(1+\sin\theta)$

$\Rightarrow \frac{1}{r}\cdot\frac{dr}{d\theta} = \frac{\cos\theta}{1+\sin\theta}$

$\Rightarrow \cot\phi_2 = \frac{\cos^2\theta/2 - \sin^2\theta/2}{\sin^2\theta/2 + \cos^2\theta/2 + 2\cos\theta/2\cdot\sin\theta/2}$

$\Rightarrow \cot\phi_2 = \frac{\cos\theta/2 - \sin\theta/2}{\cos\theta/2 + \sin\theta/2}$

$\Rightarrow \cot \phi_2 = \dfrac{1-\tan\theta/2}{1+\tan\theta/2}$ (on ÷ num & denom. by $\cos\theta/2$)

$\Rightarrow \cot \phi_2 = \tan(\pi/4 - \theta/2)$

$\Rightarrow \cot \phi_2 = \cot(\pi/2 - \pi/4 + \theta/2) = \cot(\pi/4 + \theta/2) \Rightarrow \phi_2 = \pi/4 + \theta/2.$

Point of intersection of curves.

$\Rightarrow a(1-\sin\theta_1) = a(1+\sin\theta_2)$

$\Rightarrow 2\sin\theta = 0$

$\Rightarrow \sin\theta = 0 \Rightarrow \theta = 0, \pi \ (n\pi)$

$\therefore$ angle between the curves $= |\phi_1 - \phi_2| = |(\pi/4 - \theta/2) - (\pi/4 + \theta/2)|$
$= \theta.$

(ii) $r^m = a^m(\sin m\theta + \cos m\theta)$

$\Rightarrow m \log r = m \log a + \log(\sin m\theta + \cos m\theta)$ [on taking log on both sides]

$\Rightarrow \dfrac{m}{r} \cdot \dfrac{dr}{d\theta} = 0 + \dfrac{m \cdot \cos m\theta - m \cdot \sin m\theta}{\sin m\theta + \cos m\theta}$ [on differentiating]

$\Rightarrow \dfrac{m}{r} \cdot \dfrac{dr}{d\theta} = \dfrac{m(\cos m\theta - \sin m\theta)}{\sin m\theta + \cos m\theta}$

$\Rightarrow \cot\phi = \dfrac{1 - \tan m\theta}{1 + \tan m\theta}$ [on ÷ num & denominator by $\cos m\theta$.]

$\Rightarrow \cot\phi = \tan(\pi/4 - m\theta)$

$\Rightarrow \cot\phi = \cot(\pi/2 - \pi/4 + m\theta)$

$\Rightarrow \phi = \pi/4 + m\theta$

Now, $p = r\sin\phi$ (Pedal Equation)

$$= r\sin(\pi/4 + m\theta)$$

$$= r[\sin\pi/4 \cos m\theta + \cos\pi/4 \sin m\theta]$$

$$= \frac{r}{\sqrt{2}}[\cos m\theta + \sin m\theta]$$

We know, that $r^m = a^m(\sin m\theta + \cos m\theta)$

$$\Rightarrow (\sin m\theta + \cos m\theta) = \frac{r^m}{a^m}$$

Substituting this in the obtained pedal equation we have.

$$p = \frac{r}{\sqrt{2}} \times \frac{r^m}{a^m}$$

$$\Rightarrow p = \frac{r^{m+1}}{\sqrt{2}\cdot a^m}$$

$$\Rightarrow \sqrt{2}a^m p = r^{m+1}$$

Q3. (i) Find the radius of curvature of the curve $a^2y = x^3 - y^3$ at the point where curve cuts the x-axis.

(ii) Prove that for the curve $r(1-\cos\theta) = 2a$, ρ^2 varies as r^3.

Sol^n (i) $a^2y = x^3 - y^2$

Radius of curvature in cartesian form is given as

$$\rho = \frac{(1+y_1^2)^{3/2}}{y_2}$$

where y_1 and y_2 are the first and second derivatives respectively

Here, $a^2 y = x^3 - y^3$

$$\Rightarrow a^2 \frac{dy}{dx} = 3x^2 - 3y^2 \frac{dy}{dx}.$$

$$\Rightarrow \frac{dy}{dx}(a^2 + 3y^2) = 3x^2$$

$$\Rightarrow \frac{dy}{dx} = y_1 = \frac{3x^2}{a^2+3y^2} \quad \text{—} (1)$$

The point where the curve cuts the x-axis $\Rightarrow y=0$

$$\Rightarrow 0 = x^3 - 0$$
$$\Rightarrow x = 0$$

$\therefore$ the required point $= (0,0)$

$$\therefore \frac{dy}{dx} = y_1 = 0.$$

Now, $$\frac{d^2y}{dx^2} = y_2 = \frac{\left[a^2 + 3y^2(6x) - 3x^2\left(0 + 6y\frac{dy}{dx}\right)\right]}{(a^2+3y^2)^2}$$

$$\Rightarrow y_2 = \frac{a^2 + 18xy^2 - 18yx^2\frac{dy}{dx}}{(a^2+3y^2)^2}$$

Substituting $x=0, y=0, \frac{dy}{dx}=0$ in the above expression, we get

$$y_2 = \frac{a^2+0+0}{(a^2)^2} = \frac{1}{a^2}.$$

$$\therefore \rho = \frac{(1+y_1^2)^{3/2}}{y_2} = \frac{(1+0)^{3/2}}{1/a^2} = a^2$$

is the required radius of curvature.

(ii) Given curve: $r(1-\cos\theta) = 2a$

To prove: $p^2 \propto r^3$

Solution: $r(1-\cos\theta) = 2a$

$$\Rightarrow \frac{dr}{d\theta}(1-\cancel{r}\cos\theta) + r\sin\theta = 0$$

$$\Rightarrow \frac{dr}{d\theta} = r_1 = \frac{-r\sin\theta}{(1-\cos\theta)} \quad \text{——} \ (1)$$

Now, $r_2 = \dfrac{d^2r}{d\theta^2} = \dfrac{-\left[(1-\cos\theta)\left(\frac{dr}{d\theta}\cdot\sin\theta + r\cos\theta\right) - r\sin\theta(\sin\theta)\right]}{(1-\cos\theta)^2}$

$$= \frac{-\left[(1-\cos\theta)(r\cos\theta + r_1\sin\theta) - r\sin^2\theta\right]}{(1-\cos\theta)^2}$$

$$= \frac{-\left[r\cos\theta - r\cos^2\theta + r_1\sin\theta - r_1\sin\theta\cos\theta - r\sin^2\theta\right]}{(1-\cos\theta)^2}$$

$$= \frac{-\left[r\cos\theta - r(\cos^2\theta + \sin^2\theta) + r_1\sin\theta - r_1\sin\theta\cos\theta\right]}{(1-\cos\theta)^2}$$

$$= \frac{-\left[r\cos\theta - r + r_1\sin\theta(1-\cos\theta)\right]}{(1-\cos\theta)^2}$$

$$= \frac{-\left[-r(1-\cos\theta) + r_1\sin\theta(1-\cos\theta)\right]}{(1-\cos\theta)^2}$$

$$= \frac{-\left[(1-\cos\theta)(r_1\sin\theta - r)\right]}{(1-\cos\theta)^2}$$

$$= \frac{r - r_1\sin\theta}{(1-\cos\theta)}$$

Substituting r_1 from (1), we get

$$r_2 = \frac{r - \sin\theta\left(\frac{-r\sin\theta}{1-\cos\theta}\right)}{(1-\cos\theta}$$

$$= \frac{r(1-\cos\theta) + r\sin^2\theta}{(1-\cos\theta)^2} = \frac{r(1+\sin^2\theta - \cos\theta)}{(1-\cos\theta)2}$$

Now,

$$\rho = \frac{(r^2+r_1^2)^{3/2}}{2r_1^2 + r^2 - 2rr_2}$$

Consider $\quad r_1^2 + r^2 = r^2 + \frac{r^2\sin^2\theta}{(1-\cos\theta)^2}$

$$= \frac{r^2(1-\cos\theta)^2 + r^2\sin^2\theta}{(1-\cos\theta)^2}$$

$$= \frac{r^2[1+\cos^2\theta - 2\cos\theta + \sin^2\theta]}{(1-\cos\theta)^2}$$

$$= \frac{r^2(1+1-2\cos\theta)}{(1-\cos\theta)^2}$$

$$= \frac{2r^2(1-\cos\theta)}{(1-\cos\theta)^2}$$

$$= \frac{2r^2}{(1-\cos\theta)}$$

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Consider $2r_1^2 + r^2 - rr_2 = \frac{2r^2\sin^2\theta}{(1-\cos\theta)^2} + r^2 - r\left[r\left(\frac{1+\sin^2\theta-\cos\theta}{(1-\cos\theta)^2}\right)\right]$

$$= \frac{2r^2\sin^2\theta}{(1-\cos\theta)^2} + r^2 - r^2\left[\left(\frac{1-\cos\theta}{(1-\cos\theta)^2}\right) + \frac{\sin^2\theta}{(1-\cos\theta)^2}\right]$$

$$= \frac{2r^2\sin^2\theta}{(1-\cos\theta)^2} + r^2 - \frac{r^2}{(1-\cos\theta)} + \frac{r^2\sin^2\theta}{(1-\cos\theta)^2}$$

$$= \frac{r^2\sin^2\theta}{(1-\cos\theta)^2} + r^2\left(\frac{\cancel{1}-\cos\theta-\cancel{1}}{(1-\cos\theta)}\right)$$

$$= \frac{r^2\sin^2\theta}{(1-\cos\theta)^2} \cancel{+} - \frac{r^2\cos\theta}{(1-\cos\theta)}$$

$$= r^2\left[\frac{\sin^2\theta - \cos\theta + \cos^2\theta}{(1-\cos\theta)^2}\right]$$

$$= \frac{r^2(1-\cos\theta)}{(1-\cos\theta)^2}$$

$$= \frac{r^2}{(1-\cos\theta}$$

Then, $\rho = \frac{\left[\frac{2r^2}{(1-\cos\theta)}\right]^{3/2}}{\frac{r^2}{(1-\cos\theta)}}$

$$= \frac{2\sqrt{2}\,r^3}{(1-\cos\theta)^{3/2}} \times \frac{(1-\cos\theta)}{r^2}$$

$$= \frac{2\sqrt{2}\,r}{(1-\cos\theta)^{1/2}}.$$

From the equation of curve, we have

$$r(1-\cos\theta) = 2a$$

$$(1-\cos\theta) = \frac{2a}{r}$$

$$(1-\cos\theta)^{1/2} = \left(\frac{2a}{r}\right)^{1/2}.$$

∴ Substituting this value in ρ, we get

$$\rho = \frac{2\sqrt{2}\,r}{\left(\frac{2a}{r}\right)^{1/2}}$$

$$\Rightarrow \quad \rho = \frac{2\sqrt{2}}{\sqrt{2a}}\, r^{3/2}.$$

$$\Rightarrow \quad \rho = \frac{2}{a}\, r^{3/2}$$

$$\Rightarrow \quad \rho^2 = \frac{2}{a}\, r^3$$

$$\Rightarrow \quad \rho^2 \propto r^3$$

Q4. (i) Find the rank of the matrix $A = \begin{bmatrix} 1 & 3 & -1 & 2 \\ 0 & 11 & -5 & 3 \\ 2 & -5 & 3 & 1 \\ 4 & 1 & 1 & 5 \end{bmatrix}$

(ii) Solve by Gauss - Elimination method

$$x + y + z = 9$$

$$x - 2y + 3z = 8$$

$$2x + y - z = 3$$

Anaama Singh

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Sol^n (i) $A = \begin{bmatrix} 1 & 3 & -1 & 2 \\ 0 & 11 & -5 & 3 \\ 2 & -5 & 3 & 1 \\ 4 & 1 & 1 & 5 \end{bmatrix}$

$R_3 \to R_3 - 2R_1$
$R_4 \to R_4 - 4R_1$ $\Rightarrow A = \begin{bmatrix} 1 & 3 & -1 & 2 \\ 0 & 11 & -5 & 3 \\ 0 & -11 & 5 & -3 \\ 0 & -11 & 5 & -3 \end{bmatrix}$

$R_4 \to R_4 - R_3$ $\Rightarrow A = \begin{bmatrix} 1 & 3 & -1 & 2 \\ 0 & 11 & -5 & 3 \\ 0 & -11 & 5 & -3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$R_3 \to R_3 + R_2$ $\Rightarrow A = \begin{bmatrix} 1 & 3 & -1 & 2 \\ 0 & 11 & -5 & 3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$R_2 \to R_2/11$ $\Rightarrow A = \begin{bmatrix} 1 & 3 & -1 & 2 \\ 0 & 1 & -5/11 & 3/11 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

From the row-reduced Echelon form of A, we find that $\rho(A) = 2$.

(ii) $x + y + z = 9$ $\quad x - 2y + 3z = 8$ $\quad 2x + y - z = 3$.

$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -2 & 3 \\ 2 & 1 & -1 \end{bmatrix}$ $\quad B = \begin{bmatrix} 9 \\ 8 \\ 3 \end{bmatrix}$

Anaara Singh

$$\therefore [A:B] = \begin{bmatrix} 1 & 1 & 1 & : 9 \\ 1 & -2 & 3 & : 8 \\ 2 & 1 & -1 & : 3 \end{bmatrix}$$

$R_2 \longrightarrow R_2 - R_1$
$R_3 \longrightarrow R_3 - 2R_1$

$$\Rightarrow [A:B] = \begin{bmatrix} 1 & 1 & 1 & : 9 \\ 0 & -3 & 2 & : -1 \\ 0 & -1 & -3 & : -15 \end{bmatrix}$$

$R_3 \longrightarrow 3R_3 - R_2$

$$\Rightarrow [A:B] = \begin{bmatrix} 1 & 1 & 1 & : 9 \\ 0 & -3 & 2 & : -1 \\ 0 & 0 & -11 & : -44 \end{bmatrix}$$

$R_2 \longrightarrow -R_2/3$
$R_3 \longrightarrow R_3/4$

$$\Rightarrow [A:B] = \begin{bmatrix} 1 & 1 & 1 & : 9 \\ 0 & 1 & -2/3 & : 1/3 \\ 0 & 0 & 1 & : 4 \end{bmatrix}$$

From the ~~redu~~ row-reduced Echelon form ~~of~~, we get

$x + y + z = 9$ — (I) $\quad y - 2/3 z = 1/3$ — (II) $\quad z = 4$ — (III)

From (III), (II) $\Rightarrow$ $y - \frac{4}{3} = 1/3$

$\Rightarrow y = 5/3$ — (IV)

From (III) & (IV), we have $x + \frac{5}{3} + 4 = 9$

$x = 9 - 4 - 5/3$

$x = 5 - 5/3 = 10/3$.

$\therefore x = 10/3$, $y = 5/3$, $z = 4$ ~~is~~ are the required solutions

Ahaana Singh

Q5. Solve the following system of equations by Gauss-Seidel Method.

$x + y + 54z = 110$

$27x + 6y - z = 85$ Perform 4 iterations.

$6x + 15y + 2z = 72$.

Sol^n. On rearranging, we have

$$27x + 6y - z = 85 \quad \text{— (I)}$$
$$6x + 15y + 2z = 72 \quad \text{— (II)}$$
$$x + y + 54z = 110 \quad \text{— (III)}$$

$$\Rightarrow x = \frac{1}{27}[85 - 6y + z]$$
$$y = \frac{1}{15}[72 - 6x - 2z]$$
$$z = \frac{1}{54}[110 - x - y]$$

Let $x_0 = y_0 = z_0 = 0$ be the initial condition.

1st approximation, $x_1 = \frac{1}{27}(85 - 6y_0 + z_0) = \frac{85}{27} = 3.1481$

$$y_1 = \frac{1}{15}(72 - 6x_1 - 2z_0)$$
$$= \frac{1}{15}(72 - 18.8886) = \frac{53.1114}{15} = 3.5408$$

$$z_1 = \frac{1}{54}(110 - x_1 - y_1) = \frac{103.3111}{54} = 1.9132$$

Second approximation, $x_2 = \frac{1}{27}(85 - 6y_1 + z_1) = \frac{65.6684}{27}$
$= 2.4322$

$$y_2 = \frac{1}{15}(72 - 6x_2 - 2z_1) = \frac{53.5804}{15} = 3.5720$$

$$z_2 = \frac{1}{54}(110 - x_2 - y_2) = \frac{103.9958}{54} = 1.9258$$

Dhaara Singh

3rd approximation, $x_3 = \frac{1}{27}(85 - 6y_2 - z_2) = \frac{61.6422}{27} = 2.2830$

$$y_3 = \frac{1}{15}(72 - 6x_3 - 2z_2) = \frac{54.4504}{15} = 3.6300$$

$$z_3 = \frac{1}{54}(110 - x_3 - y_3) = \frac{104.0870}{54} = 1.9275$$

4th approximation, $x_4 = \frac{1}{27}(85 - 6y_3 - z_3) = \frac{61.2925}{27} = 2.2701$

$$y_4 = \frac{1}{15}(72 - 6x_4 - 2z_3) = \frac{54.5244}{15} = 3.6350$$

$$z_4 = \frac{1}{54}(110 - x_4 - y_4) = \frac{104.0949}{54} = 1.9277.$$

$\therefore$ $x \approx 2.2701$ $y \approx 3.6350$ $z \approx 1.9277$
are the required solutions.

Q6. ~~The largest Eo~~ Find the largest Eigen value and corresponding Eigen vector of the matrix $\begin{bmatrix} 4 & 1 & -1 \\ 2 & 3 & -1 \\ -2 & 1 & 5 \end{bmatrix}$ by Rayleigh's power method. Use $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ as initial eigen vector. Perform 5 iterations.

Sol^n $Y_0 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ $A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 3 & -1 \\ -2 & 1 & 5 \end{bmatrix}$

$\therefore Y_0^{(0)} = AY_0 = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 3 & -1 \\ -2 & 1 & 5 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ -2 \end{bmatrix} = \lambda_1 A_0$

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$$Y^{(1)} = AY_0 = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 3 & -1 \\ -2 & 1 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ -2 \end{bmatrix} = \begin{bmatrix} 8 \\ 10 \\ -10 \end{bmatrix} = 8 \begin{bmatrix} 1 \\ 1.25 \\ -1.25 \end{bmatrix} = \lambda_1 Y_1$$

$$Y^{(2)} = AY_1 = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 3 & -1 \\ -2 & 1 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ 1.25 \\ -1.25 \end{bmatrix} = \begin{bmatrix} 6.5 \\ 7 \\ -7 \end{bmatrix} = 6.5 \begin{bmatrix} 1 \\ 1.0769 \\ -1.0769 \end{bmatrix}$$

$$= \lambda_2 Y_2.$$

$$Y^{(3)} = AY_2 = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 3 & -1 \\ -2 & 1 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ 1.0769 \\ -1.0769 \end{bmatrix} = \begin{bmatrix} 6.1538 \\ 6.3076 \\ -6.3076 \end{bmatrix} = 6.1538 \begin{bmatrix} 1 \\ 1.0250 \\ -1.0250 \end{bmatrix}$$

$$= \lambda_3 Y_3$$

$$Y^{(4)} = AY_3 = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 3 & -1 \\ -2 & 1 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ 1.0250 \\ -1.0250 \end{bmatrix} = \begin{bmatrix} 6.0500 \\ 6.1000 \\ -6.1000 \end{bmatrix} = 6.0500 \begin{bmatrix} 1 \\ 1.0083 \\ -1.0083 \end{bmatrix}$$

$$= \lambda_4 Y_4$$

$$Y^{(5)} = AY_4 = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 3 & -1 \\ -2 & 1 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ 1.0083 \\ -1.0083 \end{bmatrix} = \begin{bmatrix} 6.0166 \\ 6.0332 \\ -6.0332 \end{bmatrix} = 6.0166 \begin{bmatrix} 1 \\ 1.0027 \\ -1.0027 \end{bmatrix}$$

$$= \lambda_5 Y_5$$

$\therefore$ After 5 interations, Largest Eigen value = ~~6.3581~~ 6.0166

Largest Eigen vector = $\cancel{\begin{bmatrix} 1 \\ 0.9802 \\ -1.2817 \end{bmatrix}}$

$$= \begin{bmatrix} 1 \\ 1.0027 \\ -1.0027 \end{bmatrix}$$

Ahaana Singh

Q1. In the differential geometry of curves, the evolute of the curve is the locus of all its centres of curvature. This is to say that when the centre of curvature of each point on a curve is drawn, the resultant shape will be the evolute of that curve. Equivalently, an evolute is the envelope of the normal to a curve. An evolute is the locus of centers of curvature (the envelope) of a plane curve's normal.

For the astroid, $x^{2/3} + y^{2/3} = a^{2/3}$ whose parametric equations are $x = a\cos^3 t$, $y = a\sin^3 t$, show that the evolute is the curve.

$$(x+y)^{2/3} + (x-y)^{2/3} = a^{2/3}.$$

Sol^n

$x = a\cos^3 t$, $y = a\sin^3 t$

$$\frac{dx}{dt} = -3a\sin t\cos^2 t$$

$$\frac{dy}{dt} = 3a\cos t\sin^2 t$$

$$\therefore \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{3a\cos t\sin^2 t}{-3a\sin t\cos^2 t} = -\tan t = y_1$$

$$\frac{d^2y}{dx^2} = -\sec^2 t\,\frac{dt}{dx} = \frac{-\sec^2 t}{-3a\sin t\cos^2 t} = \frac{1}{3a\sin t\cos^4 t} = y_2$$

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$$\bar{x} = x - \frac{y_1(1+y_1^2)}{y_2}$$

$$= a\cos^3 t + \frac{\tan t\,(1+\tan^2 t)}{\frac{1}{3a\cos^4 t\sin t}}$$

$$= a\cos^3 t + \frac{\sin t}{\cos t}\cdot\sec^2 t\cdot 3a\sin t\cos^4 t$$

$$= a\cos^3 t + \frac{\sin t}{\cos^3 t}\cdot 3a\cos^4 t\cdot\sin t$$

$$= a\cos^3 t + 3a\cos t\sin^2 t \quad \text{—— (I)}$$

$$\bar{y} = y + \frac{1+y_1^2}{y_2}.$$

$$= a\sin^3 t + \frac{1+\tan^2 t}{\frac{1}{3a\sin t\cos^4 t}}$$

$$= a\sin^3 t + 3a\sin t\cos^4 t\sec^2 t$$

$$= a\sin^3 t + 3a\sin t\cos^2 t \quad \text{—— (II)}$$

Now, $(\bar{x}+\bar{y}) = a(\sin^3 t+\cos^3 t) + 3a(\sin t\cos t)(\sin t+\cos t)$

$$= a(\cos t+\sin t)^3 \quad \text{—— (III)}$$

$(\bar{x}-\bar{y}) = a(\sin^3 t-\cos^3 t) + 3a\sin t\cos t(\sin t-\cos t)$

$$= a(\cos t-\sin t)^3 \quad \text{—— (IV)}$$

$\therefore (\bar{x}+\bar{y})^{2/3} = a^{2/3}(\cos t+\sin t)^2$ —— (V)

$(\bar{x}-\bar{y})^{2/3} = a^{2/3}(\cos t-\sin t)^2$ —— (VI)

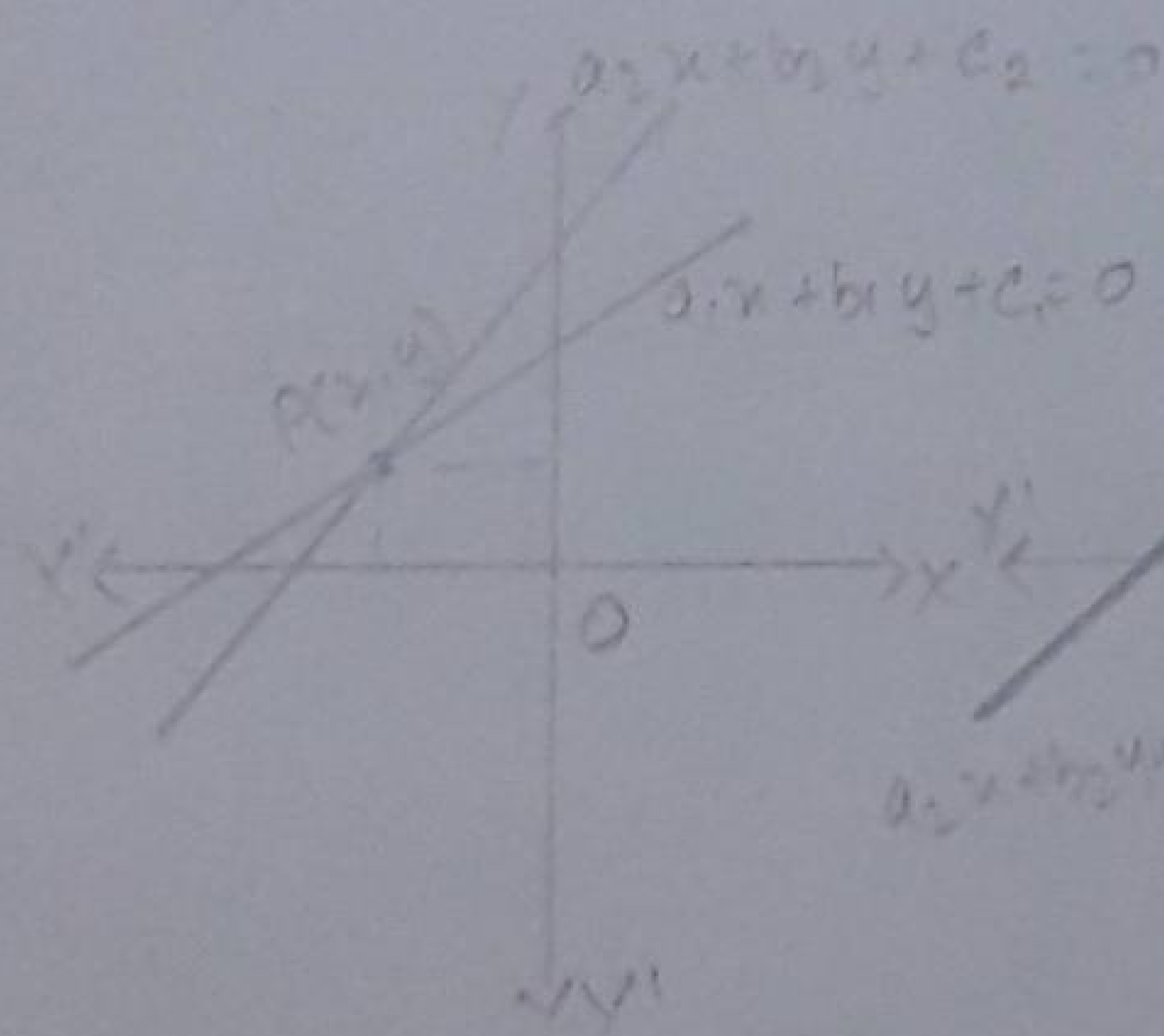
Adding (V) and (VI), we get

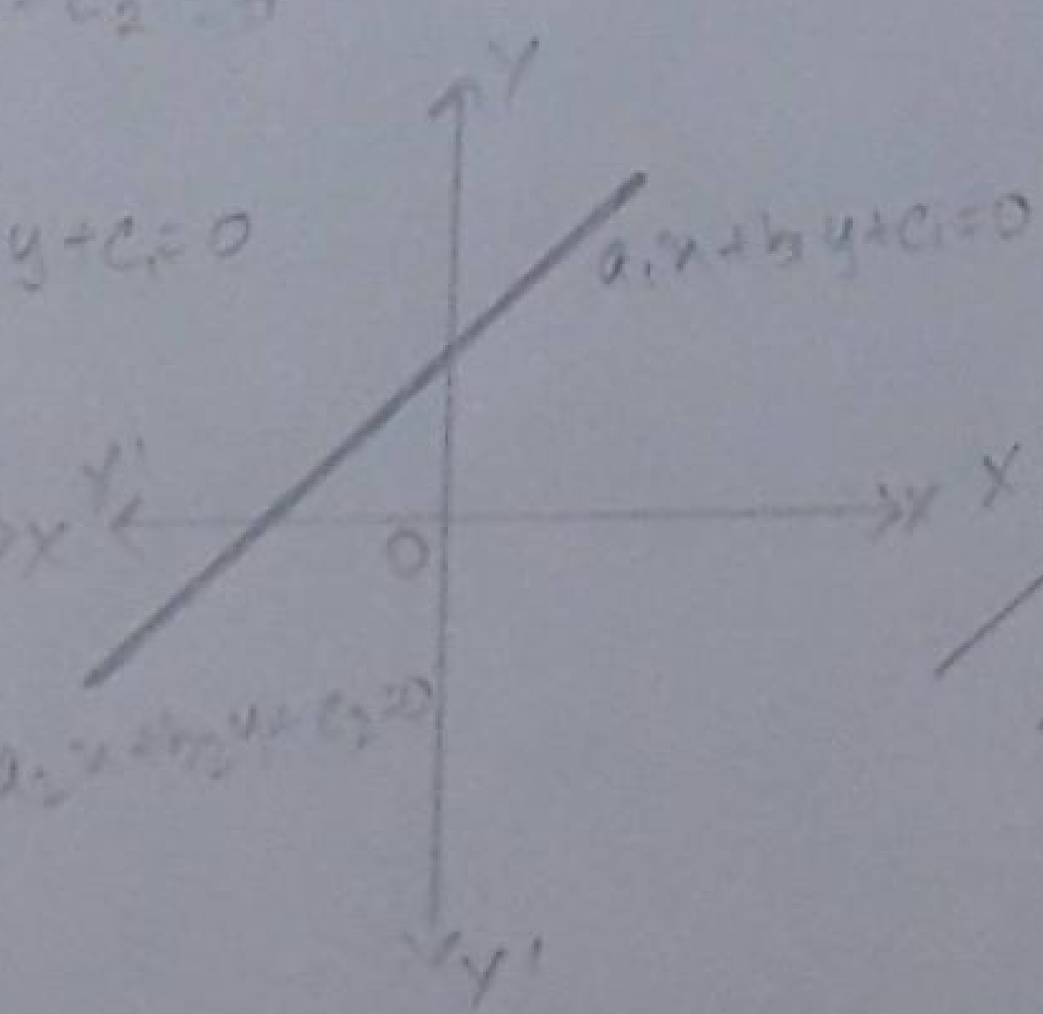
$$(\bar{x}+\bar{y})^{2/3} + (\bar{x}-\bar{y})^{2/3} = a^{2/3}\left[(\cos t+\sin t)^2 + (\cos t-\sin t)^2\right]$$

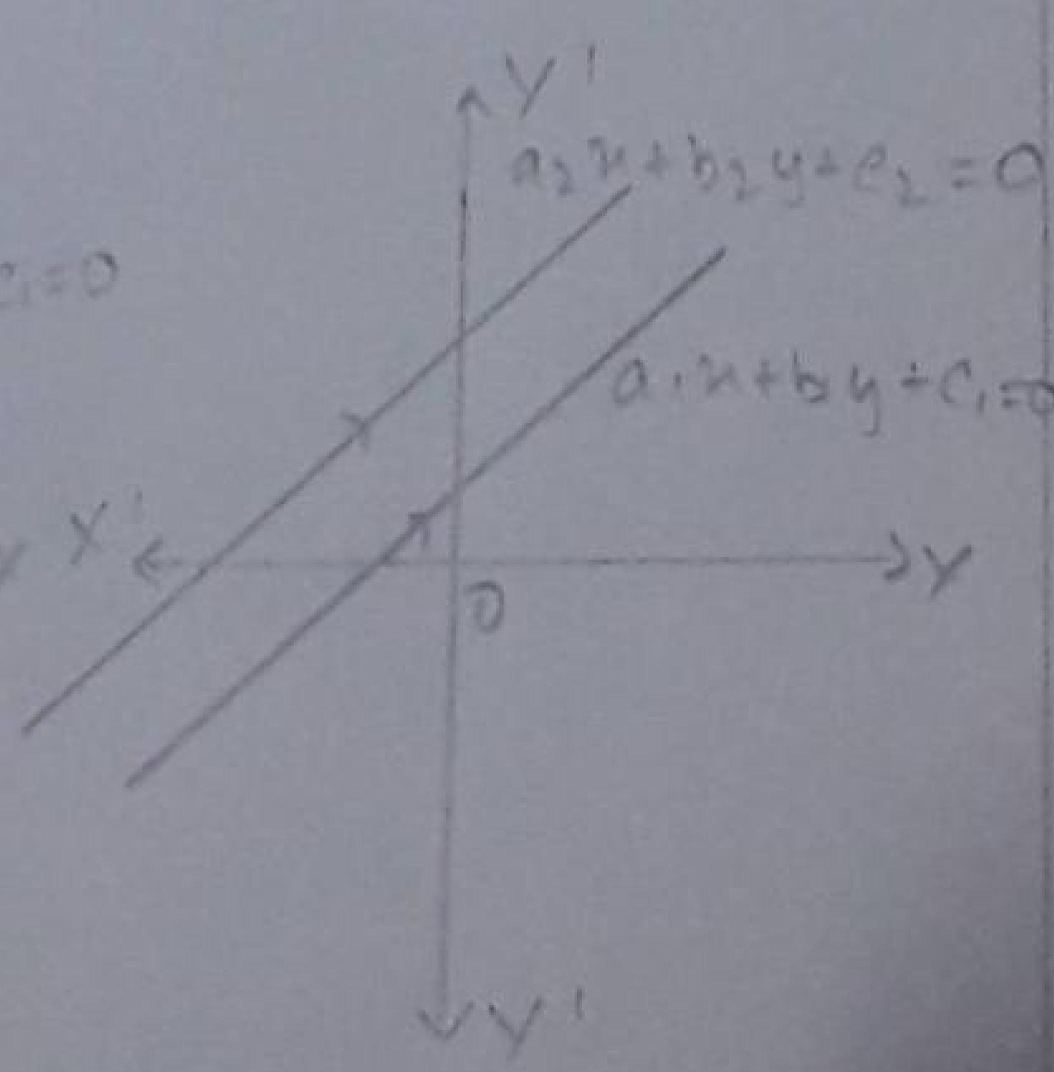
$$\Rightarrow (\bar{x}+\bar{y})^{2/3} + (\bar{x}-\bar{y})^{2/3} = 2a^{2/3}.$$

$\therefore (x+y)^{2/3} + (x-y)^{2/3} = 2a^{2/3}$ is the locus required.

Q8. A linear or non-linear system of equations is called consistent if there is atleast one set of values for the unknowns that satisfies each equation in the system. In contrast, a linear or non-linear equation system is called inconsistent if there is no set of values for the unknowns that satisfy all of the equations. If both the lines intersect at a point, then, the pair of linear equations is said to be consistent. Let these lines coincide with each other, then, the pair of linear equations is said to be dependent and consistent. Let both the lines to be parallel to each other, then there exists no solution, because the lines never intersect.







Find the values of λ an μ for which the system

$$x+y+z=6$$
$$x+2y+3z=10$$
$$x+2y+\lambda z=\mu$$

has (i) a unique solution

(ii) infinitely many solutions

(iii) no solutions

Sol^n.

$$[A:B] = \begin{bmatrix} 1 & 1 & 1 & : & 6 \\ 1 & 2 & 3 & : & 10 \\ 1 & 2 & \lambda & : & \mu \end{bmatrix}$$

$R_2 \to R_2 - R_1$

$R_3 \to R_3 - R_1$

$$[A:B] = \begin{bmatrix} 1 & 1 & 1 & : & 6 \\ 0 & 1 & 2 & : & 4 \\ 0 & 1 & \lambda-1 & : & \mu-6 \end{bmatrix}$$

$R_3 \to R_3 - R_2$

$$[A:B] = \begin{bmatrix} 1 & 1 & 1 & : & 6 \\ 0 & 1 & 2 & : & 4 \\ 0 & 0 & \lambda-3 & : & \mu-10 \end{bmatrix}$$

$\therefore \rho[A] = \rho[A:B]$

(i) The system has a unique solution if $\rho[A] = \rho[A:B] = 3$ (no. of unknowns)

For that, $\lambda - 3 \neq 0 \Rightarrow \lambda \neq 3$. the system has a unique solution.

(ii) The system has infinitely many solutions if

$\rho[A] = \rho[A:B] < 3$ (no. of unknowns)

For $\lambda = 3$ and $\mu = 10$, the system has infinitely many solutions

(iii) The system has no solution if $\rho[A] \neq \rho[A:B]$

For ~~$\lambda \neq$~~ $\lambda = 3$ and $\mu \neq 10$, the system has no solution.

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MATH CLASS TEST

Ans 1 $f(x) = \log(1+\cos x)$

$$f'(x) = \frac{-\sin x}{1+\cos x} = -\frac{\cancel{2}\sin x/2 \cos x/2}{\cancel{2}\cos^2 x/2} = -\tan x/2$$

$$\frac{C_{2\theta}}{1+C} = \frac{C^2 - S^2}{2C^2}$$

$$f''(x) = \cancel{\cos(1+\cos x)} - \sec^2\frac{x}{2} \times \frac{1}{2}$$

$$f'''(x) = -\frac{1}{\cancel{2}} \times \cancel{2}\sec\frac{x}{2} \times \sec\frac{x}{2} \times \tan\frac{x}{2} \times \frac{1}{2} = -\frac{1}{2}\sec^2\frac{x}{2}\tan\frac{x}{2}$$

$$f^{IV}(x) = -f'' \times \frac{f'}{2} = -\left[f'''\frac{f'}{2} + \frac{f''f''}{2} \times \frac{1}{2}\right]$$

$$= -\frac{1}{2}\left[2\sec\frac{x}{2}\sec\frac{x}{2}\tan^2\frac{x}{2} \times \frac{1}{2} + \sec^2\frac{x}{2}\sec^2\frac{x}{2} \times \frac{1}{2}\right]$$

$$= -\frac{1}{2}\left[\sec^2\frac{x}{2}\tan^2\frac{x}{2} + \frac{1}{2}\sec^4\frac{x}{2}\right]$$

$$f(0) = \log 2, \quad f'(0) = 0, \quad f''(0) = -\frac{1}{2}, \quad f'''(0) = 0, \quad f^{IV}(0) = -\frac{1}{4}$$

$$f(x) = f(0) + \frac{x}{1!}f'(0) + \frac{x^2}{2!}f''(0) + \frac{x^3}{3!}f'''(0) + \frac{x^4}{4!}f^{IV}(0) + \cdots$$

$$= \log 2 + x \times 0 + \frac{x^2}{2} \times -\frac{1}{2} + \frac{x^3}{6} \times 0 + \frac{x^4}{24} \times -\frac{1}{4} + \cdots$$

$$\boxed{f(x) = \log 2 - \frac{x^2}{4} - \frac{x^4}{96} + \cdots}$$

Ans 2. $A = f(x) = \lim_{x\to 0}\left\{\frac{a^x + b^x + c^x}{3}\right\}^{1/x}$

applying log:

$$\log A = \lim_{x\to 0}\frac{1}{x}\log\left(\frac{a^x+b^x+c^x}{3}\right) \cdots \frac{0}{0}$$

$$= \lim_{x\to 0}\frac{\log\left(\frac{a^x+b^x+c^x}{3}\right)}{x} \cdots \frac{0}{0}$$

differentiating:

$$= \lim_{x\to 0}\frac{\cancel{3}}{a^x+b^x+c^x} \times \frac{a^x\log a + b^x\log b + c^x\log c}{\cancel{3}}$$

applying $\lim_{x\to 0}$ & substituting $x=0$,

$$= \frac{\log a + \log b + \log c}{3} = \log(abc)^{1/3}$$

$$\boxed{\therefore A = (abc)^{1/3}}$$

P.T.O

Ans 3. $u = f\left(\frac{x}{y}, \frac{y}{z}, \frac{z}{x}\right)$

$\frac{x}{y} = r, \quad \frac{y}{z} = s, \quad \frac{z}{x} = t.$

$u = f(r, s, t).$

$$\boxed{\frac{\partial u}{\partial x} = \frac{\partial u}{\partial r} \times \frac{\partial r}{\partial x} + \frac{\partial u}{\partial s} \times \frac{\partial s}{\partial x} + \frac{\partial u}{\partial t} \times \frac{\partial t}{\partial x}}$$

$$\frac{\partial r}{\partial x} = \frac{1}{y} \qquad \frac{\partial r}{\partial y} = -\frac{x}{y^2} \qquad \frac{\partial r}{\partial z} = 0$$

$$\frac{\partial s}{\partial x} = 0 \qquad \frac{\partial s}{\partial y} = \frac{1}{z} \qquad \frac{\partial s}{\partial z} = -\frac{y}{z^2}$$

$$\frac{\partial t}{\partial x} = -\frac{z}{x^2} \qquad \frac{\partial t}{\partial y} = 0 \qquad \frac{\partial t}{\partial z} = \frac{1}{x}$$

$$\therefore \frac{\partial u}{\partial x} = \frac{\partial u}{\partial r} \times \frac{1}{y} + \frac{\partial u}{\partial s} \times 0 + \frac{\partial u}{\partial t} \times -\frac{z}{x^2}$$

$$\frac{\partial u}{\partial x} = \frac{1}{y}\frac{\partial u}{\partial r} - \frac{z}{x^2}\frac{\partial u}{\partial t}$$

$$\boxed{x\frac{\partial u}{\partial x} = \frac{x}{y}\frac{\partial u}{\partial r} - \frac{z}{x}\frac{\partial u}{\partial t}} \quad - (1)$$

$$\frac{\partial u}{\partial y} = \frac{\partial u}{\partial r} \times \frac{\partial r}{\partial y} + \frac{\partial u}{\partial s} \times \frac{\partial s}{\partial y} + \frac{\partial u}{\partial t} \times \frac{\partial t}{\partial y}$$

$$= \frac{\partial u}{\partial r} \times -\frac{x}{y^2} + \frac{\partial u}{\partial s} \times \frac{1}{z} + 0$$

$$\boxed{y\frac{\partial u}{\partial y} = -\frac{x}{y}\frac{\partial u}{\partial r} + \frac{y}{z}\frac{\partial u}{\partial s}} \quad - (2)$$

$$\frac{\partial u}{\partial z} = \frac{\partial u}{\partial r} \times \frac{\partial r}{\partial z} + \frac{\partial u}{\partial s} \times \frac{\partial s}{\partial z} + \frac{\partial u}{\partial t} \times \frac{\partial t}{\partial z}$$

$$= \frac{\partial u}{\partial r} \times 0 + \frac{\partial u}{\partial s} \times -\frac{y}{z^2} + \frac{\partial u}{\partial t} \times \frac{1}{x}$$

$$\boxed{z\frac{\partial u}{\partial z} = -\frac{y}{z}\frac{\partial u}{\partial s} + \frac{z}{x}\frac{\partial u}{\partial t}} \quad - (3)$$

P.T.O

from ①, ② & ③

$$\Rightarrow x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = \frac{x}{y}\cancel{\frac{\partial u}{\partial r}} - \frac{z}{x}\cancel{\frac{\partial u}{\partial t}} + \frac{y}{z}\cancel{\frac{\partial u}{\partial s}} - \frac{x}{y}\cancel{\frac{\partial u}{\partial r}} + \frac{z}{x}\cancel{\frac{\partial u}{\partial t}} - \frac{y}{z}\cancel{\frac{\partial u}{\partial s}}$$

$$\boxed{\therefore\ x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 0}$$

Ans $\frac{\partial(u,v,w)}{\partial(x,y,z)}$, $u = x^2+y^2+z^2$, $v = xy+yz+zx$, $w = x+y+z$.

$\frac{\partial u}{\partial x} = 2x \quad \frac{\partial u}{\partial y} = 2y \quad \frac{\partial u}{\partial z} = 2z$

$\frac{\partial v}{\partial x} = y+z \quad \frac{\partial v}{\partial y} = x+z \quad \frac{\partial v}{\partial z} = y+x$

$\frac{\partial w}{\partial x} = 1 \quad \frac{\partial w}{\partial y} = 1 \quad \frac{\partial w}{\partial z} = 1$

$$\frac{\partial(u,v,w)}{\partial(x,y,z)} = \begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} & \frac{\partial u}{\partial z} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} & \frac{\partial v}{\partial z} \\ \frac{\partial w}{\partial x} & \frac{\partial w}{\partial y} & \frac{\partial w}{\partial z} \end{vmatrix} = \begin{vmatrix} 2x & 2y & 2z \\ y+z & x+z & y+x \\ 1 & 1 & 1 \end{vmatrix}$$

expanding determinant.

$$\Rightarrow 2x(\cancel{x}+z-y-\cancel{x}) - 2y(\cancel{y}+z-\cancel{y}-x) + 2z(y+\cancel{z}-x-\cancel{z})$$

$$\Rightarrow 2xz - 2xy - 2yz + 2xy + 2yz - 2zx$$

$$\Rightarrow 0$$

$$\boxed{\therefore\ \frac{\partial(u,v,w)}{\partial(x,y,z)} = 0}$$

Paper - 2

Ans 1. $f(x) = \log(\sec x + \tan x)$

$$f'(x) = \frac{\sec x \tan x + \sec^2 x}{\sec x + \tan x} = \frac{\sin x + 1}{\cos x(1+\sin x)} = \sec x.$$

$f'(0) = 1.$

$f''(x) = \sec x \tan x \Rightarrow f''(0) = 0$

$f'''(x) = \sec x \tan^2 x + \sec^3 x \Rightarrow f'''(0) = 1$

$f^{IV}(x) = \sec x \tan^3 x + \sec x \times 2\tan x \sec^2 x + 3\sec^2 x \times \sec x \tan x$

$\Rightarrow f^{IV}(0) = 0$

$$f(x) = f(0) + \frac{x}{1!}f'(0) + \frac{x^2}{2!}f''(0) + \frac{x^3}{3!}f'''(0) + \frac{x^4}{4!}f^{IV}(0) + \dots$$

$$f(x) = 1 + 0 + \frac{x^2}{2}\times 1 + 0 + \dots$$

Ans 2. $$A = \lim_{x\to 0}(\cos x)^{1/x^2}$$

$$\log A = \lim_{x\to 0}\frac{1}{x^2}\log(\cos x) = \lim_{x\to 0}\frac{\log(\cos x)}{x^2} \Rightarrow \lim_{x\to 0}\frac{-\tan x}{2x}$$

$$= \lim_{x\to 0}\frac{-\sec^2 x}{2} = -\frac{1}{2}$$

$$A = e^{-1/2} = \frac{1}{\sqrt{e}}$$

Ans 3. $z = xy^2 + x^2y$, $x = at$ $y = 2at$

$$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial x}\times\frac{\partial x}{\partial t} + \frac{\partial z}{\partial y}\times\frac{\partial y}{\partial t}$$

$= (y^2 + 2xy)\times a + (2xy + x^2)2a$

$= ay^2 + 2axy + 2axy + 2ax^2$

$= 2ax^2 + ay^2 + 4axy$

$= 2a \times a^2t^2 + a\times 4a^2t^2 + 4a\times at\times 2at$

$= 2a^3t^2 + 4a^3t^2 + 8a^3t^2 = 14a^3t^2$

Ans 4. $u = x + 3y^2 - z^3$, $v = 4x^2yz$, $w = 2z^2 - xy$.
$(1,-1,0)$

$\frac{\partial u}{\partial x} = 1$ $\quad$ $\frac{\partial u}{\partial y} = 6y = -6$ $\quad$ $\frac{\partial u}{\partial z} = -3z^2 = 0$

$\frac{\partial v}{\partial x} = 8xyz = 0$ $\quad$ $\frac{\partial v}{\partial y} = 4x^2 z = 0$ $\quad$ $\frac{\partial v}{\partial z} = \cancel{4z = 0} \; 4x^2y = -4$

$\frac{\partial w}{\partial x} = -y = 1$ $\quad$ $\frac{\partial w}{\partial y} = -x = -1$ $\quad$ $\frac{\partial w}{\partial z} = 4z = 0.$

$$\frac{\partial(u,v,w)}{\partial(x,y,z)} = \begin{vmatrix} 1 & -6 & 0 \\ 0 & 0 & -4 \\ 1 & -1 & 0 \end{vmatrix}$$

$$= -4 + (6\times +4) + 0 = 24 - 4 = \underline{\underline{20}}$$

Paper-3

Ans 1. $f(x) = \sqrt{1 + \sin 2x}$

$= \sin x + \cos x \Rightarrow f(0) = 1$

$f'(x) = \cos x - \sin x \Rightarrow f'(0) = 1$

$f''(x) = -\sin x - \cos x \Rightarrow f''(0) = -1$

$f'''(x) = -\cos x + \sin x \Rightarrow f'''(0) = -1$

$f^{iv}(x) = +\sin x + \cos x \Rightarrow f^{iv}(0) = 1$

$$f(x) = 1 + \frac{x}{1!} - \frac{x^2}{2} - \frac{x^3}{6} + \frac{x^4}{24} + \cdots$$

Ans 2. $u = f(y-z, z-x, x-y)$

$r = y - z$ $\quad$ $s = z - x$ $\quad$ $t = x - y$

$\frac{\partial r}{\partial x} = 0$ $\quad$ $\frac{\partial s}{\partial x} = -1$ $\quad$ $\frac{\partial t}{\partial x} = 1$

$\frac{\partial r}{\partial y} = 1$ $\quad$ $\frac{\partial s}{\partial y} = 0$ $\quad$ $\frac{\partial t}{\partial y} = -1$

$\frac{\partial r}{\partial z} = -1$ $\quad$ $\frac{\partial s}{\partial z} = 1$ $\quad$ $\frac{\partial t}{\partial z} = 0$

$$\frac{\partial u}{\partial x} = \frac{\partial u}{\partial r} \times \frac{\partial r}{\partial x} + \frac{\partial u}{\partial s} \times \frac{\partial s}{\partial x} + \frac{\partial u}{\partial t} \times \frac{\partial t}{\partial x}$$

$$= \frac{\partial u}{\partial r} \times 0 + \frac{\partial u}{\partial s} \times -1 + \frac{\partial u}{\partial t} \times 1 = -\frac{\partial u}{\partial s} + \frac{\partial u}{\partial t}$$

$$\frac{\partial u}{\partial y} = \frac{\partial u}{\partial r}\times\frac{\partial r}{\partial y} + \frac{\partial u}{\partial s}\times\frac{\partial s}{\partial y} + \frac{\partial u}{\partial t}\times\frac{\partial t}{\partial y}$$

$$= \frac{\partial u}{\partial r}\times 1 + \frac{\partial u}{\partial s}\times 0 + \frac{\partial u}{\partial t}\times -1 = \frac{\partial u}{\partial r} - \frac{\partial u}{\partial t}$$

$$\frac{\partial u}{\partial z} = \frac{\partial u}{\partial r}\times\frac{\partial r}{\partial z} + \frac{\partial u}{\partial s}\times\frac{\partial s}{\partial z} + \frac{\partial u}{\partial t}\times\frac{\partial t}{\partial z}$$

$$= \frac{\partial u}{\partial r}\times -1 + \frac{\partial u}{\partial s}\times 1 + \frac{\partial u}{\partial t}\times 0 = -\frac{\partial u}{\partial r} + \frac{\partial u}{\partial s}$$

$$\therefore \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0.$$

Ans 3. $u = \frac{yz}{x}$ $\quad v = \frac{zx}{y}$ $\quad w = \frac{xy}{z}$

$$\frac{\partial u}{\partial x} = -\frac{yz}{x^2} \qquad \frac{\partial v}{\partial x} = \frac{z}{y} \qquad \frac{\partial w}{\partial x} = \frac{y}{z}$$

$$\frac{\partial u}{\partial y} = \frac{z}{x} \qquad \frac{\partial v}{\partial y} = -\frac{zx}{y^2} \qquad \frac{\partial w}{\partial y} = \frac{x}{z}$$

$$\frac{\partial u}{\partial z} = \frac{y}{x} \qquad \frac{\partial v}{\partial z} = \frac{x}{y} \qquad \frac{\partial w}{\partial z} = -\frac{xy}{z^2}$$

$$\frac{\partial(u,v,w)}{\partial(x,y,z)} = \begin{vmatrix} -\frac{yz}{x^2} & \frac{z}{y} & \frac{y}{z} \\ \frac{z}{x} & -\frac{zx}{y^2} & \frac{x}{z} \\ \frac{y}{x} & \frac{x}{y} & -\frac{xy}{z^2} \end{vmatrix}$$

$$= -\frac{yz}{x^2}\left[\frac{x^2yz}{y^2z^2} - \frac{x}{yz}\right] - \frac{z}{y}\left[-\frac{y}{z} - \frac{y}{z}\right] + \frac{y}{z}\left[\frac{zx}{xy} + \frac{z}{y}\right]$$

$$= -1 + 1 + 2 + 2 = \underline{\underline{4}}$$

Ans 4. $$A = \lim_{x\to a}\left(2 - \left(\frac{x}{a}\right)\right)^{\tan\left(\frac{\pi x}{2a}\right)}$$

$$\Rightarrow \log A = \lim_{x\to a}\tan\left(\frac{\pi x}{2a}\right)\log\left(2-\frac{x}{a}\right) = \lim_{x\to a}\frac{\log\left(2-\frac{x}{a}\right)}{\cot\left(\frac{\pi x}{2a}\right)}$$

$$= \lim_{x\to a}\frac{-1/2a-x}{-\operatorname{cosec}^2\left(\frac{\pi x}{2a}\right)\times\frac{\pi}{2a}} = \frac{+1/a}{\pi/2a} = \frac{2}{\pi}$$

$$\boxed{\therefore A = e^{2/\pi}}$$

Paper 4

Ans 1. $f(x) = \log(1+x)$

$f(0) = 0$

$f'(x) = 1/1+x$, $f'(0) = 1$

$f''(x) = -1/(1+x)^2$, $f''(0) = -1$

$f'''(x) = +2/(1+x)^3$, $f'''(0) = 2$

$f^{iv}(x) = -6/(1+x)^4$, $f^{iv}(0) = -6$.

$$f(x) = 0 + x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots$$

Ans 2. $A = \lim_{x\to0}\left(\frac{a^x+b^x}{2}\right)^{1/x} \Rightarrow \log A = \lim_{x\to0}\frac{1}{x}\log\left(\frac{a^x+b^x}{2}\right)$

$$\log A = \lim_{x\to0}\frac{\log\left(\frac{a^x+b^x}{2}\right)}{x} = \lim_{x\to0}\frac{\frac{2}{a^x+b^x}\times\left(\frac{a^x\log a+b^x\log b}{2}\right)}{1}$$

$\log A = \log(ab)^{1/2}$

$\Rightarrow A = (ab)^{1/2}$

Ans 3. $u = e^{x^2+y^2+z^2}$, $t=0$, $x = t^2+1$, $y = t\cos t$, $z = \sin t$

$$\frac{du}{dt} = \frac{du}{dx}\times\frac{dx}{dt} + \frac{du}{dy}\times\frac{dy}{dt} + \frac{du}{dz}\times\frac{dz}{dt}$$

$$= 2xe^{x^2+y^2+z^2}\times 2t + 2ye^{x^2+y^2+z^2}\times(\cos t - t\sin t) + 2ze^{x^2+y^2+z^2}\times\cos t$$

$$= 0 + 2ye^{x^2+y^2+z^2} + 2ze^{x^2+y^2+z^2}$$

$$\Rightarrow 2\times t\cos t\times e^{x^2+y^2+z^2} + 2\times\sin t\times e^{x^2+y^2+z^2}$$

$$\boxed{\therefore \frac{du}{dt} = 0}$$

Ans 4. $u = x+y+z$, $v = y+z$, $w = z$

$\frac{\partial u}{\partial x} = 1$, $\frac{\partial v}{\partial x} = 0$, $\frac{\partial v}{\partial x} = 0$

$\frac{\partial u}{\partial y} = 1$, $\frac{\partial v}{\partial y} = 1$, $\frac{\partial v}{\partial y} = 0$

$\frac{\partial u}{\partial z} = 1$, $\frac{\partial v}{\partial z} = 1$, $\frac{\partial v}{\partial z} = 1$

$$J = \begin{vmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{vmatrix}$$

$$\boxed{\Rightarrow J = 1}$$

Mathematics Assignment

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ISE - D
Roll. no: 03.

1. Evaluate: (a) $\int_{-c}^{c}\int_{-b}^{b}\int_{-a}^{a}(x^2+y^2+z^2)\,dz\,dy\,dx$.

Soln: Let, $I = \int_{-c}^{c}\int_{-b}^{b}\int_{-a}^{a}(x^2+y^2+z^2)\,dz\,dy\,dx = \int_{-c}^{c}\int_{-b}^{b} x^2z+y^2z+\frac{z^3}{3}\Big|_{-a}^{a}\,dy\,dx$

$I = \int_{-c}^{c}\int_{-b}^{b} x^2(a+a)+y^2(a+a)+\frac{1}{3}(a^3+a^3)\,dy\,dx = \int_{-c}^{c}\int_{-b}^{b} 2ax^2+2ay^2+\frac{2a^3}{3}\,dy\,dx$

$I = \int_{-c}^{c} 2ax^2y+\frac{2ay^3}{3}+\frac{2a^3y}{3}\Big|_{-b}^{b}\,dx = \int_{-c}^{c} 2ax^2(b+b)+\frac{2a}{3}(b^3+b^3)+\frac{2a^3}{3}(b+b)\,dx.$

$I = \int_{-c}^{c} 4ax^2b+\frac{4ab^3}{3}+\frac{4a^3b}{3}\,dx = \frac{4ax^3b}{3}+\frac{4ab^3x}{3}+\frac{4a^3bx}{3}\Big|_{-c}^{c}$

$I = \frac{4ab}{3}(c^3+c^3)+\frac{4ab^3}{3}(c+c)+\frac{4a^3b}{3}(c+c) = \frac{8abc^3}{3}+\frac{8ab^3c}{3}+\frac{8a^3bc}{3}$

$\therefore I = \frac{8abc}{3}(a^2+b^2+c^2)$

(b) $\int_{-1}^{1}\int_{0}^{z}\int_{x-z}^{x+z}(x+y+z)\,dy\,dx\,dz$

Soln:- Let, $I = \int_{-1}^{1}\int_{0}^{z}\int_{x-z}^{x+z}(x+y+z)\,dy\,dx\,dz = \int_{-1}^{1}\int_{0}^{z} xy+\frac{y^2}{2}+zy\Big|_{x-z}^{x+z}\,dx\,dz$

$I = \int_{-1}^{1}\int_{0}^{z} x(\cancel{x}+z-\cancel{x}+z)+\frac{1}{2}\left[(x+z)^2-(x-z)^2\right]+z[\cancel{x}+z-\cancel{x}+z]\,dx\,dz$

$I = \int_{-1}^{1}\int_{0}^{z} 2xz+\frac{1}{\cancel{2}}(\cancel{4}xz)+2z^2\,dx\,dz$

$I = \int_{-1}^{1}\int_{0}^{z} 4xz+2z^2\,dx\,dz$

$$I = \int_{-1}^{1}\int_{0}^{2} 4xz + 2z^2 \, dx \, dz$$

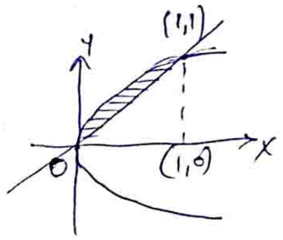
$$I = \int_{-1}^{1} \frac{4x^2z}{2} + 2z^2x \Big|_0^2 dz = \int_{-1}^{1} 2z(z^2) + 2z^2(z) \, dz$$

$$I = \int_{-1}^{1} 2z^3 + 2z^3 \, dz = \frac{2z^4}{4} + \frac{2z^4}{4}\Big|_{-1}^{1} = \frac{z^4}{2} + \frac{z^4}{2}\Big|_{-1}^{1}$$

$$I = \frac{2z^4}{2}\Big|_{-1}^{1} = z^4\Big|_{-1}^{1} = (1-1) = 0.$$

$\therefore \quad I = 0.$

Q. Evaluate $\int_0^1 \int_x^{\sqrt{x}} xy \, dy \, dx$ by change of order of integration.



Soln-

$y = x$

$y = \sqrt{x} \Rightarrow x = y^2$

$$I = \int_{y=0}^{1} \int_{x=y^2}^{y} xy \, dx \, dy$$

$$= \frac{1}{2}\int_0^1 x^2y \Big|_{y^2}^{y} dy = \frac{1}{2}\int_0^1 y^3 - y^5 \, dy$$

$$= \frac{1}{2}\left[\frac{y^4}{4} - \frac{y^6}{6}\right]_0^1$$

$$= \frac{1}{2}\left[\frac{1}{4} - \frac{1}{6}\right] = \frac{1}{2}\left[\frac{2}{24}\right]$$

$\therefore \quad I = \frac{1}{24}$ sq.units.

3. Evaluate $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)}\,dx\,dy$ by changing into polar coordinates.

Solⁿ:- Let, $I = \int_0^\infty \int_0^\infty e^{-(x^2+y^2)}\,dx\,dy$

WKT, $x = r\cos\theta$, $y = r\sin\theta$, $x^2+y^2 = r^2$, $dx\,dy = r\,dr\,d\theta$.

$r \longrightarrow 0$ to ∞ and $\theta \longrightarrow 0$ to $\pi/2$

$$I = \int_0^{\pi/2} \int_0^\infty e^{-r^2}\, r\,dr\,d\theta$$

Let, $r^2 = t$

$2r\,dr = dt$

$r\cdot dr = \frac{dt}{2}$

$$I = \int_0^{\pi/2} \int_0^\infty e^{-t} \cdot \frac{dt}{2} \cdot d\theta = \frac{1}{2} \int_0^{\pi/2} \frac{e^{-t}}{-1}\Big|_0^\infty d\theta = \frac{1}{2} \int_0^{\pi/2} -\left(e^{\infty} - e^{0}\right) d\theta$$

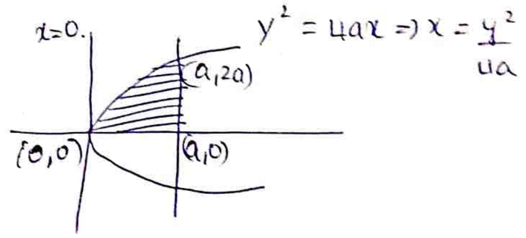
$$I = \frac{1}{2} \int_0^{\pi/2} 1\,d\theta = \frac{1}{2} \times \theta \Big|_0^{\pi/2} = \frac{1}{2} \times \frac{\pi}{2} = \frac{\pi}{4}$$

$$\therefore\ I = \frac{\pi}{4}$$

4. Evaluate by changing the order of integration $\int_0^a \int_a^{2\sqrt{ax}} x^2\,dy\,dx$, $a>0$

Solⁿ:- $y = 2\sqrt{ax}$

$y^2 = 4ax$



On changing the order of integration,

$$I = \int_{y=0}^{2a} \int_{x=\frac{y^2}{4a}}^{a} x^2\,dx\,dy$$

$$I = \frac{1}{3} \int_0^{2a} x^3 \Big|_{y^2/4a}^{a} dy$$

$$= \cancel{\frac{2a}{3}}\ \frac{1}{3} \int_0^{2a} a^3 - \left(\frac{y^2}{4a}\right)^3 dy$$

$$I = \frac{1}{3}\int_0^{2a} a^3 - \frac{y^6}{64a^3}\,dy = \frac{1}{3}\int_0^{2a} a^3 y - \frac{1}{64a^3}\times\frac{y^7}{7}$$

$$= \frac{1}{3}\left[a^3(2a) - \frac{1}{64a^3}\times\frac{1}{7}\times(2a)^7\right]$$

$$= \frac{1}{3}\left[2a^4 - \frac{1}{448a^3}\times 128a^7\right]$$

$$= \frac{1}{3}\left[2a^4 - \frac{2}{7}a^4\right]$$

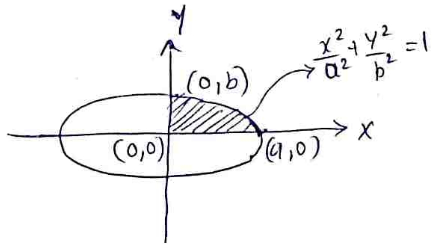
$$= \frac{2}{3}\left[a^4 - \frac{a^4}{7}\right] = \frac{2}{3}a^4\left[1-\frac{1}{7}\right] = \frac{2}{3}\times\frac{6}{7}a^4$$

$$\therefore\ I = \frac{4}{7}a^4 \text{ sq.units.}$$

5. Find the area bounded by the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ by double integration.

Soln. –

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$



$$\frac{y^2}{b^2} = \frac{a^2-x^2}{a^2} \Rightarrow y = \frac{b}{a}\sqrt{a^2-x^2}$$

$$\text{Area} = \iint_A dx\,dy \qquad \begin{array}{l} x \rightarrow 0 \text{ to } a \\ y \rightarrow 0 \text{ to } \frac{b}{a}\sqrt{a^2-x^2}\end{array}$$

$$\text{Area of ellipse} = 4\int_{x=0}^{a}\int_{y=0}^{b/a\sqrt{a^2-x^2}} dy\,dx$$

$$= \frac{4}{\cancel{2}}\int_0^a \cancel{y^2}\Big|_0^{\frac{b}{a}\sqrt{a^2-x^2}} dx$$

$$= \frac{4b}{a}\int_0^a \sqrt{a^2-x^2}\,dx = \frac{4b}{a}\left[\frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right)\right]_0^a$$

$$= \frac{4b}{a}\times\frac{a^2}{2}\sin^{-1}(1) = \frac{4b}{a}\times\frac{a^2}{2}\times\frac{\pi}{2}$$

$$\therefore\ \text{Area of ellipse} = \pi ab \text{ sq. units.}$$

6. Find the volume of the tetrahedron bounded by the planes : $x=0$, $y=0$, $z=0$ & $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$.

Sol^n:- $\frac{z}{c} = 1-\frac{x}{a}-\frac{y}{b}$ $\therefore z = c\left(1-\frac{x}{a}-\frac{y}{b}\right)$

put $z=0$, $0=\left(1-\frac{x}{a}-\frac{y}{b}\right) \Rightarrow \frac{y}{b} = 1-\frac{x}{a}$ $\therefore y = b\left(1-\frac{x}{a}\right)$

If $z=0, y=0$, $\frac{x}{a}=1$ $\therefore \boxed{x=a}$

$\therefore x \to 0$ to a & $y \to 0$ to $b\left(1-\frac{x}{a}\right)$

$$\text{volume} = \int_{x=0}^{a}\int_{y=0}^{b(1-\frac{x}{a})} c\left(1-\frac{x}{a}-\frac{y}{b}\right) dy\, dx = c\int_0^a \left. y-\frac{x}{a}y-\frac{y^2}{2b}\right|_0^{b(1-x/a)} dx$$

$$= c\int_0^a b\left(1-\frac{x}{a}\right) - \frac{xb}{a}\left(1-\frac{x}{a}\right) - \frac{b}{2}\left(1-\frac{x}{a}\right)^2 dx$$

$$= c\int_0^a b\left(1-\frac{x}{a}\right)\left\{1-\frac{x}{a}-\frac{1}{2}\left(1-\frac{x}{a}\right)\right\} dx = c\int_0^a b\left(1-\frac{x}{a}\right)\frac{1}{2}\left(1-\frac{x}{a}\right) dx$$

$$= \frac{bc}{2}\int_0^a \left(1-\frac{x}{a}\right)^2 dx = \frac{bc}{2}\left[-\frac{a}{3}\left(1-\frac{x}{a}\right)^3\right]_0^a$$

$$= -\frac{abc}{6}(0-1) = \frac{abc}{6} \text{ cubic units.}$$

7. Solve $xyp^2-(x^2+y^2)p+xy=0$.

Sol^n:- Here, $a=xy$, $b=-(x^2+y^2)$, $c=xy$

$$p = \frac{-b\pm\sqrt{b^2-4ac}}{2a} = \frac{(x^2+y^2)\pm\sqrt{(x^2+y^2)^2+4x^2y^2}}{2xy}$$

$$p = \frac{(x^2+y^2)\pm\sqrt{(x^2)^2+(y^2)^2+2x^2y^2+4x^2y^2}}{2xy} = \frac{(x^2+y^2)\pm\sqrt{(x^2)^2+(y^2)^2-2x^2y^2}}{2xy}$$

$$p = \frac{(x^2+y^2)\pm(x^2-y^2)}{2xy}$$

$$p = \frac{x^2+y^2+x^2-y^2}{2xy} \qquad p = \frac{x^2+y^2-x^2+y^2}{2xy}$$

$$p = \frac{2x^2}{2xy} = \frac{x}{y} \qquad p = \frac{2y^2}{2xy} = \frac{y}{x}$$

$$\frac{dy}{dx} = \frac{x}{y} \qquad \frac{dy}{dx} = \frac{y}{x}$$

$$y \cdot dy = x \cdot dx \qquad \frac{dy}{y} = \frac{dx}{x}$$

$$\frac{y^2}{2} = \frac{x^2}{2} + c \qquad \log y = \log x + \log c$$

$$\log y = \log(cx)$$

$$y^2 - x^2 - 2c = 0 \qquad y = cx \Rightarrow y - cx = 0.$$

$\therefore$ General solⁿ is,

$$(y^2-x^2-2c)(y-cx) = 0.$$

9. Solve $p^2 + 2py\cot x = y^2$

Solⁿ:- $a = 1$, $b = 2y\cot x$, $c = -y^2$

$$p = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-2y\cot x \pm \sqrt{4y^2\cot^2 x + 4y^2}}{2}$$

$$p = \frac{-2y\cot x \pm 2y\sqrt{1+\cot^2 x}}{2} = \frac{-y\cot x \pm y\operatorname{cosec} x}{1}$$

$$p = -y\cot x + y\operatorname{cosec} x \qquad p = -y\cot x - y\operatorname{cosec} x$$

$$p = y(\operatorname{cosec} x - \cot x) \qquad p = -y(\cot x + \operatorname{cosec} x)$$

$$\frac{dy}{dx} = y(\operatorname{cosec} x - \cot x) \qquad \frac{dy}{dx} = -y(\cot x + \operatorname{cosec} x)$$

$$\frac{dy}{y} = (\operatorname{cosec} x - \cot x)\,dx \qquad \frac{dy}{-y} = (\cot x + \operatorname{cosec} x)\,dx$$

$$\frac{dy}{y} = \left(\frac{1-\cos x}{\sin x}\right)dx \qquad +\frac{dy}{y} = -\left(\frac{1+\cos x}{\sin x}\right)dx$$

$$\frac{dy}{y} = \tan x/2 \; dx \qquad \frac{dy}{y} = -\cot x \; dx$$

$$\log y = \log(\sec^2 x/2) + c \qquad \log y = \log(\operatorname{cosec}^2 x/2) + c$$

$$y = \sec^2 x/2 \cdot c \qquad y = \operatorname{cosec}^2 x/2 \cdot c$$

$\therefore$ General solⁿ is, $(y - c\sec^2 x/2)(y - \operatorname{cosec}^2 x/2 \cdot c) = 0.$

9. Solve the equation $(px-y)(py+x) = 2p$ by reducing into clairaut's form taking the substitution $X = x^2$, $Y = y^2$?

Soln:- $\frac{dX}{dx} = 2x$ $\qquad \frac{dY}{dy} = 2y$

$$p = \frac{dy}{dx} = \frac{dy}{dY} \cdot \frac{dY}{dX} \cdot \frac{dX}{dx} = \frac{1}{2y} \cdot P \cdot 2x = \frac{x}{y} P$$

$$p = \frac{\sqrt{X}}{\sqrt{Y}} P$$

consider, $(px-y)(py+x) = 2p$

$$\left(\frac{\sqrt{X}}{\sqrt{Y}} P\sqrt{X} - \sqrt{Y}\right)\left(\frac{\sqrt{X}}{\sqrt{Y}} P\sqrt{Y} + \sqrt{X}\right) = 2\frac{\sqrt{X}}{\sqrt{Y}} P$$

$$\left(\frac{PX-Y}{\cancel{\sqrt{Y}}}\right)\left(\cancel{\sqrt{X}}(P+1)\right) = \frac{2\cancel{\sqrt{X}} P}{\cancel{\sqrt{Y}}}$$

$$(PX-Y)(P+1) = 2P$$

$$\cancel{PX - Y = 2P - P - 1}$$

$$PX - Y = \frac{2P}{P+1}$$

$$Y = PX + \frac{2P}{P+1}$$

This is in the clairaut's form & hence general soln is,

$$Y = CX - \frac{2C}{C+1}$$

$\therefore$ Required G.S is, $y^2 = cx^2 - \frac{2c}{c+1}$

10. If the temperature of the air is 30°C and a metal ball cools from 100°c to 70°c in 15 minutes. Find how long will it take for metal ball to reach a temperature of 40°C.

Soln:- Let, 'θ' be the temperature of the body at any time 't'.

By Newton's law of cooling,

$$\frac{d\theta}{dt} = -K(\theta - \theta_0)$$

The soln is, $\theta(t) = \theta_0 + Ce^{-Kt}$

Given: $\theta_0 = 30°C$.

$\theta(t) = 30° + Ce^{-Kt}$

At $\theta = 100°C$, when $t = 0$.

$100 = 30 + C$

$\therefore \boxed{C = 70°C}$

At $t = 15$ min, $\theta = 70°C$

$70 = 30 + 70e^{-15K}$

$40 = 70e^{-15K}$

$\frac{4}{7} = e^{-15K}$

$\log\left(\frac{4}{7}\right) = -15K$

$15K = \log\left(\frac{7}{4}\right)$

$K = \frac{\log(7/4)}{15}$

$\therefore \boxed{K = 0.0373}$

* At ~~0~~ any time, temp is given by,

$\theta(t) = 30 + 70e^{-0.0373t}$

At $\theta = 40°C$,

$40 = 30 + 70e^{-0.0373t}$

$10 = 70e^{-0.0373t}$

$\frac{1}{7} = e^{-0.0373t}$

$\log\left(\frac{1}{7}\right) = -0.0373t$

$t = \frac{\log(7)}{0.0373}$

$\therefore \boxed{t = 52.169 \text{ min.}}$

11. Find the orthogonal trajectories of the family of the curves

(a) $\frac{x^2}{a^2} dx + \frac{y^2}{b^2+\lambda} dy = 1$

Soln:-

$\frac{x^2}{a^2} + \frac{y^2}{b^2+\lambda} = 1 \Rightarrow \frac{-y^2}{b^2+\lambda} = \frac{x^2-a^2}{a^2}$ — (1).

Dwt. x,

$\frac{2x}{a^2} + \frac{2y}{b^2+\lambda} \cdot \frac{dy}{dx} = 0$

$\frac{y}{b^2+\lambda} \cdot \frac{dy}{dx} = \frac{-x}{a^2}$ — (2)

$\frac{(2)}{(1)} \Rightarrow \dfrac{\frac{y}{b^2+\lambda} \cdot \frac{dy}{dx} = \frac{-x}{a^2}}{\frac{-y^2}{b^2+\lambda} = \frac{x^2-a^2}{a^2}} \Rightarrow \frac{1}{y} \cdot \frac{dy}{dx} = \frac{x}{x^2-a^2}$

$$\frac{1}{y}\cdot\frac{dy}{dx} = \frac{x}{x^2-a^2}$$

Change $\frac{dy}{dx}$ to $\frac{-dx}{dy}$

$$\frac{1}{y}\;\frac{-dx}{dy} = \frac{x}{x^2-a^2}$$

$$-\left(\frac{x^2-a^2}{x}\right)dx = y\cdot dy$$

$$\left(\frac{a^2}{x}-x\right)dx = y\cdot dy$$

Integrate on both sides.

$$a^2\log x - \frac{x^2}{2} = \frac{y^2}{2}+c$$

$$\therefore\ x^2-y^2 = -2a^2\log x + c$$

(b) $r^n \cos n\theta = a^n$

Soln:- $n\log r + \log(\cos n\theta) = n\log a$

Dwt. θ

$$\frac{n}{r}\frac{dr}{d\theta} + \frac{-n\sin n\theta}{\cos n\theta} = 0$$

$$\frac{n}{r}\cdot\frac{dr}{d\theta} = \frac{n\sin n\theta}{\cos n\theta}$$

$$\frac{1}{r}\cdot\frac{dr}{d\theta} = \tan n\theta$$

Change $\frac{dr}{d\theta}$ to $\left(-r^2\, d\theta/dr\right)$

$$\frac{1}{r}\left(-r^2\frac{d\theta}{dr}\right) = \tan n\theta$$

$$-r\cdot\frac{d\theta}{dr} = \tan n\theta$$

$$\cot n\theta = \frac{-1}{r}\cdot dr$$

Integrate

$$\frac{\log(\sin n\theta)}{n} = -\log r + \log c$$

$$\log(\sin n\theta) = -n\log r + n\log c$$

$$\log(\sin n\theta) + \log r^n = \log c^n$$

$$\log\left(r^n \sin n\theta\right) = \log c^n$$

$$\therefore\ r^n \sin n\theta = c^n$$

(c) $y^2 = 4ax$

Soln:- $\frac{y^2}{x} = 4a$

$$\frac{x\left(2y \cdot \frac{dy}{dx}\right) - y^2}{x^2} = 0$$

$$2xy \cdot \frac{dy}{dx} - y^2 = 0$$

$$2x \cdot \frac{dy}{dx} - y = 0.$$

change $\frac{dy}{dx}$ to $-\frac{dx}{dy}$

$$2x\left(-\frac{dx}{dy}\right) = y$$

$$-2x \cdot dx = y \cdot dy$$

Integrate

$$-2\frac{x^2}{2} = \frac{y^2}{2} + c$$

$$-x^2 = \frac{y^2}{2} + c$$

$\therefore$ $2x^2 + y^2 = 2c$ is the required solution.

12. Prove that $\beta(m,n) = \frac{\Gamma m \Gamma n}{\Gamma m+n}$

Soln:- By definition, $\Gamma n = \int_0^\infty e^{-x} x^{n-1} dx$

Put $x = t^2$

$dx = 2t\,dt$

$$\Gamma n = 2\int_0^\infty e^{-t^2} t^{2n-2} t' dt$$

$$\Gamma n = 2\int_0^\infty e^{-t^2} t^{2n-1} dt$$

Consider, $\Gamma m \Gamma n = \left\{2\int_0^\infty e^{-x^2} x^{2m-1} dx\right\}\left\{2\int_0^\infty e^{-y^2} y^{2n-1} dy\right\}$

$$\Gamma m \Gamma n = 4\int_0^\infty\int_0^\infty e^{-(x^2+y^2)} x^{2m-1} y^{2n-1} dx\,dy$$

$x = r\cos\theta$, $y = r\sin\theta$, $x^2+y^2=r^2$, $dx\,dy = r\,dr\,d\theta$, $r \to 0$ to ∞, $\theta \to 0$ to $\pi/2$

$$\Gamma m \Gamma n = 4\int_0^{\pi/2}\int_0^\infty e^{-r^2} (r\cos\theta)^{2m-1} (r\sin\theta)^{2n-1} r\,dr\,d\theta$$

$$\Gamma m \Gamma n = \left\{2\int_0^\infty e^{-r^2} r^{2m+2n-2} r\,dr\right\}\left\{2\int_0^{\pi/2} \cos^{2m-1}\theta \cdot \sin^{2n-1}\theta\,d\theta\right\}$$

$$\Gamma m \Gamma n = 2\int_0^\infty e^{-r^2} (r^2)^{m+n-1} r\,dr\} \cdot \beta(m,n)$$

Put $r^2 = t$

$2r \cdot dr = dt$

$r \cdot dr = \frac{dt}{2}$

$$\Gamma m \Gamma n = \frac{2}{2} \int_0^\infty e^{-t} \cdot t^{m+n-1} \, dt \cdot \beta(m,n)$$

$$\Gamma m \Gamma n = \int_0^\infty e^{-t} \, t^{m+n-1} \, dt \cdot \beta(m,n)$$

$$\Gamma m \Gamma n = \Gamma m+n \cdot \beta(m,n)$$

$$\therefore \beta(m,n) = \frac{\Gamma m \Gamma n}{\Gamma m+n}$$

13. Show that $\displaystyle\int_0^{\pi/2} \frac{d\theta}{\sqrt{\sin\theta}} \times \int_0^{\pi/2} \sqrt{\sin\theta} \, d\theta = \pi$

Soln:- Let, $I = \displaystyle\int_0^{\pi/2} \sin^{-1/2}\theta \cdot d\theta \times \int_0^{\pi/2} \sin^{1/2}\theta \, d\theta$

WKT, $\displaystyle\int_0^{\pi/2} \sin^p\theta \, d\theta = \frac{\Gamma\frac{p+1}{2}}{\Gamma\frac{p+2}{2}} \cdot \frac{\sqrt{\pi}}{2}$

$$I = \frac{\Gamma\frac{-1/2+1}{2}}{\Gamma\frac{-1/2+2}{2}} \frac{\sqrt{\pi}}{2} \times \frac{\Gamma\frac{1/2+1}{2}}{\Gamma\frac{1/2+2}{2}} \frac{\sqrt{\pi}}{2}$$

$$I = \frac{\pi}{4} \left\{ \frac{\Gamma 1/4}{\cancel{\Gamma 3/4}} \times \frac{\cancel{\Gamma 3/4}}{\Gamma\frac{5}{4}} \right\} = \frac{\pi}{4} \frac{\Gamma\frac{1}{4}}{\Gamma\frac{5}{4}} = \frac{\pi}{4} \frac{\Gamma\frac{1}{4}}{\Gamma\frac{1}{4}+1}$$

$$I = \frac{\pi}{4} \frac{\cancel{\Gamma\frac{1}{4}}}{\frac{1}{4}\cancel{\Gamma\frac{1}{4}}} = \frac{\pi}{4} \times 4$$

$$\therefore I = \pi$$

14. A series circuit with resistance R, inductance L and electromotive force E is governed by the differential eqn $L\frac{di}{dt} + Ri = E$, where L and R are constants & initially the current is zero. Find the current at any time 't'.

Soln:- Given; $L\cdot\frac{di}{dt} + Ri = E$

$$\frac{di}{dt} + \frac{R}{L}i = \frac{E}{L}$$

Soln is given by,

$$i\,e^{\int\frac{R}{L}dt} = \int \frac{E}{L}e^{\int\frac{R}{L}\cdot dt}\,dt + c$$

$$i\,e^{\frac{Rt}{L}} = \frac{E}{L}\int e^{\frac{Rt}{L}}\,dt + c$$

$$ie^{\frac{Rt}{L}} = \frac{E\times L}{L}\,\frac{e^{\frac{Rt}{L}}}{R} + c$$

$$ie^{\frac{Rt}{L}} = \frac{E}{R}e^{\frac{Rt}{L}} + c$$

$\times^{ly}$ by $e^{-\frac{Rt}{L}}$

$$i = \frac{E}{R} + ce^{-\frac{Rt}{L}} \quad \text{--- (1)}$$

At $t=0$, $i=0$.

$$(1) \Rightarrow 0 = \frac{E}{R} + c \Rightarrow c = -\frac{E}{R} \quad \text{--- (2)}$$

(2) in (1),

$$i = \frac{E}{R} - \frac{E}{R}e^{-\frac{Rt}{L}}$$

$$\therefore i = \frac{E}{R}\left(1 - e^{-Rt/L}\right)$$

15. Show that $\displaystyle\int_0^\infty \frac{e^{-x^2}\,dx}{\sqrt{x}} \times \int_0^\infty \sqrt{x}\, e^{-x^2}\,dx = \frac{\pi}{2\sqrt{2}}$

Sol^n:- Let, $I_1 = \displaystyle\int_0^\infty x^{-1/2}\cdot e^{-x^2}\,dx$

Put $x^2 = t$

$x = t^{1/2}$

$dx = \dfrac{1}{2t^{1/2}}\,dt$

$I_1 = \dfrac{1}{2}\displaystyle\int_0^\infty t^{-1/4-1/2}\,e^{-t}\,dt = \dfrac{1}{2}\int_0^\infty e^{-t}\cdot t^{-1/4}\,dt$

Here, $n-1 = -3/4$

$\therefore\ n = 1/4$

$\therefore\ I_1 = \dfrac{1}{2}\Gamma\dfrac{3}{4}$ —(1)

$I_2 = \displaystyle\int_0^\infty x^{1/2}\cdot e^{-x^2}\,dx$

$x^2 = t$

$x = t^{1/2}$

$dx = \dfrac{1}{2t^{1/2}}\,dt$

$I_2 = \dfrac{1}{2}\displaystyle\int_0^\infty t^{1/4-1/2}\,e^{-t}\,dt$

Here, $n-1 = -1/4$

$\therefore\ n = \Gamma\dfrac{3}{4}$

$\therefore\ I_2 = \dfrac{1}{2}\Gamma\dfrac{3}{4}$ —(2).

$I = I_1 \times I_2$

$= \dfrac{1}{2}\Gamma\dfrac{1}{4} \times \dfrac{1}{2}\Gamma\dfrac{3}{4}$

$= \dfrac{1}{4}\Gamma\dfrac{1}{4}\,\Gamma\dfrac{3}{4}$

$= \dfrac{1}{4}\Gamma\dfrac{1}{4}\,\Gamma\left(1-\dfrac{1}{4}\right)$

$= \dfrac{1}{4}\,\dfrac{\pi}{\sin\frac{\pi}{4}}$

$I = \dfrac{1}{4}\,\dfrac{\pi}{\frac{1}{\sqrt{2}}}$

$\therefore\ I = \dfrac{\pi}{2\sqrt{2}}$

BMS INSTITUTE OF TECHNOLOGY AND MANAGEMENT

Avalahalli, Doddaballapur Main Road, Bengaluru - 560064

DEPARTMENT OF MATHEMATICS

FIRST INTERNAL ASSESSMENT TEST, JANUARY - 2021

Course Name	CALCULUS AND LINEAR ALGEBRA	**Course Code**	18MAT11
Branch & Semester	All branches & I Semester	**Max. Marks**	50
Name of the Course Coordinator (s)	Dr. CAS, Dr. AA, Dr. JJI,Dr, KS, AK, Dr. AMS, Dr. KVK, STK, Dr. BVP, Dr. SPN, Neha	**Date**	28.01.2020 (9.30 AM to 11 AM)

Answer FIVE full questions choosing 3 questions from Part A. Part B is compulsory.

Part A

1	(i) With usual notations Prove that $tan\phi = r\frac{d\theta}{dr}$ (ii) Find the angle between the curves $r = \sin\theta + cos\,\theta$ and $r = 2\,sin\theta$	(5+5) Marks	(CO2) (K3)
	OR		
2	(i) Show that the curves $r = a\,(1 - sin\theta)$ and $r = a\,(1 + sin\theta)$ are orthogonal. (ii) Find the pedal equation of the curve $r^m = a^m(sinm\theta + cosm\theta)$	(5+5) Marks	(CO1) (K3)

3	(i) Find the radius of curvature of the curve $a^2y = x^3 - y^3$ at the point where curve cuts the x-axis. (ii) Prove that for the curve $r\,(1 - cos\theta) = 2a$, ρ^2 varies as r^3.	(5+5) Marks	(CO1) (K3)
	OR		
4	(i) Find the rank of the matrix $A = \begin{bmatrix} 1 & 3 & -1 & 2 \\ 0 & 11 & -5 & 3 \\ 2 & -5 & 3 & 1 \\ 4 & 1 & 1 & 5 \end{bmatrix}$ (ii) Solve by Gauss Elimination Method $x + y + z = 9,\ \ x - 2y + 3z = 8,\ \ 2x + y - z = 3$	(5+5) Marks	(CO5) (K3)

5	Solve the following system of equations by Gauss Seidel Method $x + y + 54z = 110,\ \ 27x + 6y - z = 85,\ \ 6x + 15y + 2z = 72.$ Perform 4 iterations.	10 Marks	(CO5) (K3)
	OR		
6	Find the largest Eigen value and corresponding Eigen Vector of the matrix $\begin{bmatrix} 4 & 1 & -1 \\ 2 & 3 & -1 \\ -2 & 1 & 5 \end{bmatrix}$ by Rayleigh's power method. Use $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ as initial eigen vector. Perform 5 iterations.	10 Marks	(CO5) (K3)

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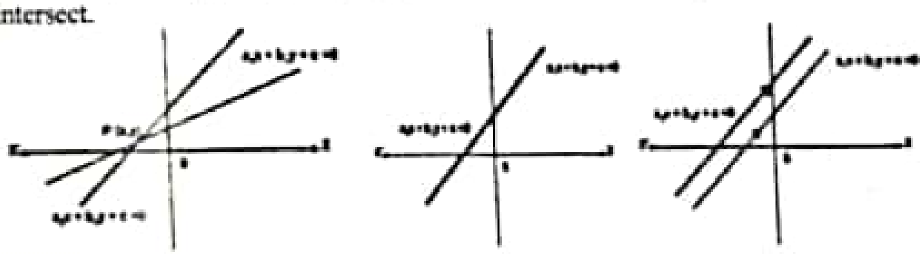
FIRST INTERNAL ASSESSMENT TEST, JANUARY - 2021

Part II

7	In the differential geometry of curves, the evolute of a curve is the locus of all its centers of curvature. That is to say that when the center of curvature of each point on a curve is drawn, the resultant shape will be the evolute of that curve. Equivalently, an evolute is the envelope of the normal to a curve. An evolute is the locus of centers of curvature (the envelope) of a plane curve's normal. For the Astroid $x^{2/3}+y^{2/3}=a^{2/3}$ whose parametric equations are $x=a\cos^3 t$, $y=a\sin^3 t$, show that the evolute is the curve $(x+y)^{2/3}+(x-y)^{2/3}=a^{2/3}$.	10 Marks	(CO1) (K4)
8	A linear or nonlinear system of equations is called consistent if there is at least one set of values for the unknowns that satisfies each equation in the system. In contrast, a linear or non linear equation system is called inconsistent if there is no set of values for the unknowns that satisfy all of the equations. If both the lines intersect at a point, then, the pair of linear equations is said to be consistent. Let these lines coincide with each other, then, the pair of linear equations is said to be dependent and consistent. Let both the lines to be parallel to each other, then there exists no solution, because the lines never intersect.  Find the values of λ and μ for which the system $x+y+z=6, x+2y+3z=10, x+2y+\lambda z=\mu$ has (a) a unique solution, (b) infinitely many solutions, (c) no solution.	10 Marks	(CO5) (K4)

Course Outcomes (COs)

Students will be able to

CO1	Apply the knowledge of calculus to solve problems related to polar curves and its applications in determining the bentness of a curve.
CO2	Learn the notion of partial differentiation to calculate rates of change of multivariate functions and solve problems related to composite functions and Jacobians.
CO3	Apply the concept of change of order of integration and variables to evaluate multiple integrals and their usage in computing the area and volumes
CO4	Solve first order linear/nonlinear differential equation analytically using standard methods
CO5	Make use of matrix theory for solving system of linear equations and compute Eigen values and eigenvectors required for matrix diagonalization process.

Bloom's Taxonomy

K1- Remembering, K2 - Understanding, K3 - Applying, K4 - Analyzing, K5 - Evaluating, K6 - Creating

T.K. Sreelakshmi	[signature]	T.K Sreelakshmi	[signature]
Course Coordinator	**Paper setter**	**Reviewer**	**Head of the Department**

BMS Institute of Technology & Management
Department of mathematics
First Internal Assessment Test. 26.01.2021. 18MAT11

(1) $\tan\phi = r\frac{d\theta}{dr}$ bookwork (5)

i) $r = \sin\theta + \cos\theta$ $\phi_1 = \frac{\pi}{4}+\theta$ (2m)

$r = 2\sin\theta$ $\phi_2 = \theta$. (2m)

Angle between curves $= \frac{\pi}{4}$ (1m)

(2) (i) $r = a(1-\sin\theta)$ $r = a(1+\sin\theta)$

$\cot\phi_1 = \frac{-\cos\theta}{1-\sin\theta}$ (1m) $\cot\phi_2 = \frac{\cos\theta}{1+\sin\theta}$ (1m)

$\cot\phi_1 \cdot \cot\phi_2 = -1$ (2m)

$\therefore \tan\phi_1 \cdot \tan\phi_2 = -1$ orthogonal (1m)

ii) $r^m = a^m[\cos m\theta + \sin m\theta]$

$\cot\phi = \cot\left[\frac{\pi}{4}+m\theta\right]$ $\therefore \phi = \frac{\pi}{4}+m\theta$ → (2)

$p = r\sin\phi = r\sin\left(\frac{\pi}{4}+m\theta\right)$ → (1)

$\therefore p = \frac{r^{m+1}}{\sqrt{2}a^m}$ ⇒ $r^{m+1} = \sqrt{2}a^m p$ → (2)

(3) (i) $a^2y = x^3 - y^3$ cuts x-axis $y=0$ ⇒ $x=0$ point $(0,0)$ (1m)

$y_1 = \frac{3x^2}{a^2+3y^2}$ at $(0,0)$ $y_1 = 0$ (1m)

$y_2 = 0$ at $(0,0)$ (2m) $\therefore$ RoC $\rho = \frac{(1+y_1^2)^{3/2}}{y_2} = \frac{1}{0} = \infty$ (1m)

(ii) $r(1-\cos\theta) = 2a$ $r_1 = -r\cot\theta/2$ (1m) $r_2 = \frac{r}{2}\csc^2\frac{\theta}{2} + r\cot^2\theta/2$ (1m)

$\rho = 2r\csc\theta/2$ (2m)

Eliminating θ proving $\rho^2 = \frac{4}{a}r^3$ $\rho^2 \propto r^3$ (1m)

4. $\begin{bmatrix} 1 & 3 & -1 & 2 \\ 0 & 11 & -5 & 3 \\ 2 & -5 & 3 & 1 \\ 4 & 1 & 1 & 5 \end{bmatrix} \begin{matrix} R_3 = R_3 - 2R_1 \\ R_4 = R_4 - 4R_1 \end{matrix} \sim \begin{bmatrix} 1 & 3 & -1 & 2 \\ 0 & 11 & -5 & 3 \\ 0 & -11 & 5 & -3 \\ 0 & -11 & 5 & -3 \end{bmatrix} \begin{matrix} R_3 = R_3 + R_2 \\ R_4 = R_4 + R_2 \end{matrix} \sim \begin{bmatrix} 1 & 3 & -1 & 2 \\ 0 & 11 & -5 & 3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ $\rho(A) = 2$ (1m)

(2m) (2m)

) $A:B = \left[\begin{array}{ccc|c} 1 & 1 & 1 & 9 \\ 1 & -2 & 3 & 8 \\ 2 & 1 & -1 & 3 \end{array}\right] \begin{matrix} R_2 = R_2 - R_1 \\ R_3 = R_3 - 2R_1 \end{matrix} \sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 9 \\ 0 & -3 & 2 & -1 \\ 0 & -1 & -3 & -15 \end{array}\right] R_3 = 3R_3 - R_2 \sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 9 \\ 0 & -3 & 2 & -1 \\ 0 & 0 & 11 & 44 \end{array}\right]$

(1m) (1m) (1m)

$\therefore\ x+y+z = 9 \qquad -3y+2z = -1 \qquad 11z = 44$

$x+3+4 = 9 \qquad -3y+8 = -1 \qquad z = 4$

$x = 2 \qquad -3y = -9$

$y = 3$ (1m)

Soln $x=2\ y=3\ z=4$ (1m)

5. Rearranging given system for diagonally dominant and setting $x\ 12$ (2m)

$x = \dfrac{85 - 6y + z}{27} \qquad y = \dfrac{72 - 6x - 2z}{15} \qquad z = \dfrac{110 - x - y}{54}$.

$\{x^0, y^0, z^0\}\ (0,0,0)$

$x^{(1)} = 3.14815$	$x^{(2)} = 2.43218$	$x^{(3)} = 2.42569$	$x^{(4)} = 2.42549$
$y^{(1)} = 3.54074$	$y^{(2)} = 3.57204$	$y^{(3)} = 3.57294$	$y^{(4)} = 3.57301$
$z^{(1)} = 1.91317$	$z^{(2)} = 1.92585$	$z^{(3)} = 1.92595$	$z^{(4)} = 1.92595$

$(2+2 \ 2+2)$ 4

6. $A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 3 & -1 \\ -2 & 1 & 5 \end{bmatrix} \quad x^{(0)} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \qquad Ax^{(0)} = 4 \begin{bmatrix} 1 \\ 0.5 \\ -0.5 \end{bmatrix} \qquad Ax^{(1)} = 5 \begin{bmatrix} 1 \\ 0.8 \\ -0.8 \end{bmatrix}$

$Ax^{(2)} = 5.6 \begin{bmatrix} 1 \\ 0.93 \\ -0.93 \end{bmatrix} \qquad Ax^{(3)} = 5.86 \begin{bmatrix} 1 \\ 0.98 \\ -0.98 \end{bmatrix} \qquad Ax^{(4)} = 5.96 \begin{bmatrix} 1 \\ 0.99 \\ -0.99 \end{bmatrix}$

2+2+2+2+2

12 m

7. $x^{2/3} + y^{2/3} = a^{2/3} \qquad x = a\cos^3 t \qquad y = a\sin^3 t \qquad \frac{dy}{dx} = -\tan t \qquad \frac{d^2y}{dx^2} = \frac{1}{3a\cos^4 t \sin t}$ (1)+(2)m

$\bar{x} = x - \frac{y_1(1+y_1^2)}{y_2} = a\cos^3 t + 3a\sin^2 t\cos t \qquad \bar{y} = y + \frac{1+y_1^2}{y_2} = a\sin^3 t + 3a\cos^2 t\sin t$ (2)+(2)

$\bar{x}+\bar{y} = a[\cos t + \sin t]^3 \qquad \bar{x}-\bar{y} = a[\cos t - \sin t]^3$ (1)+(1) m

$(\bar{x}+\bar{y})^{2/3} + (\bar{x}-\bar{y})^{2/3} = a^{2/3}$ (1)m

8. $\left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 1 & 2 & 3 & 10 \\ 1 & 2 & \lambda & \mu \end{array}\right] \begin{matrix} R_2 = R_2 - R_1 \\ R_3 = R_3 - R_1 \end{matrix} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 1 & \lambda-1 & \mu-6 \end{array}\right] R_3 = R_3 - R_2 \sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & \lambda-3 & \mu-10 \end{array}\right]$ 2+2m

Unique soln $\lambda \neq 3$ 2m

Infinite soln $\lambda = 3,\ \mu \neq 10$ 2m

No solution $\lambda = 3,\ \mu \neq 10$ 2m

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CBCS SCHEME

USN | | | | | | | | | | | **18MAT11**

First Semester B.E. Degree Examination, Jan./Feb. 2021
Calculus and Linear Algebra

Time: 3 hrs.   Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. With usual notation, prove that $\dfrac{1}{p^2} = \dfrac{1}{r^2} + \dfrac{1}{r^4}\left(\dfrac{dr}{d\theta}\right)^2$ **(06 Marks)**

b. Find the radius of curvature for the parabola $\dfrac{2a}{r} = 1 + \cos\theta$ **(06 Marks)**

c. Show that the evolute of the parabola $y^2 = 4ax$ is $27ay^2 = 4(x-2a)^3$ **(08 Marks)**

OR

2 a. Find the angle of intersection of the curves $r = 2\sin\theta$ and $r = 2\cos\theta$ **(06 Marks)**

b. Find the pedal equation of the curve $r^m = a^m [\cos m\theta + \sin m\theta]$ **(06 Marks)**

c. For the curve $y = \dfrac{ax}{a+x}$, show that $\left(\dfrac{2\rho}{a}\right)^{\frac{2}{3}} = \left(\dfrac{x}{y}\right)^2 + \left(\dfrac{y}{x}\right)^2$ **(08 Marks)**

Module-2

3 a. Using Maclaurin's series, prove that

$$\sqrt{1+\cos 2x} = \sqrt{2}\left[1 - \frac{x^2}{2} + \frac{x^4}{24} - \dots\dots\right]$$ **(06 Marks)**

b. Evaluate i) $\underset{x \to 0}{\mathrm{Lt}} \left(\dfrac{1}{x}\right)^{2\sin x}$ ii) $\underset{x \to 0}{\mathrm{Lt}} \left[\dfrac{a^x + b^x + c^x}{3}\right]^{\frac{1}{x}}$ **(07 Marks)**

c. Examine the function $f(x, y) = 2 + 2x + 2y - x^2 - y^2$ for its extreme values. **(07 Marks)**

OR

4 a. If $u = f(y-z, z-x, x-y)$ then prove that $u_x + u_y + u_z = 0$. **(06 Marks)**

b. If $u = 3x + 2y - z$; $v = x - 2y + z$; $w = x^2 + 2xy - xz$ then show that $\dfrac{\partial(u, v, w)}{\partial(x, y, z)} = 0$ **(07 Marks)**

c. The pressure P at any point (x, y, z) in space $P = 400xyz^2$. Find the highest pressure at the surface of a unit sphere $x^2 + y^2 + z^2 = 1$. **(07 Marks)**

Module-3

5 a. Evaluate: $\displaystyle\int_{-1}^{1}\int_{0}^{z}\int_{x-z}^{x+z}(x + y + z)\,dx\,dy\,dz$ **(06 Marks)**

b. Obtain the relation between Beta and Gama functions in the form $\beta(m, n) = \dfrac{\Gamma m\,\Gamma n}{\Gamma(m+n)}$ **(07 Marks)**

c. Find the centre of Gravity of the curve $r = a(1 + \cos\theta)$. **(07 Marks)**

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

OR

6 a. Change the order of integration and evaluate $\int_0^1 \int_{\sqrt{y}}^1 dx\,dy$ **(06 Marks)**

b. A Pyramid is bounded by three coordinate planes and the plane $x + 2y + 3z = 6$. Compute the volume by double integration. **(07 Marks)**

c. Prove that $\int_0^{\pi/2} \sqrt{\sin\theta}\, d\theta \times \int_0^{\pi/2} \frac{d\theta}{\sqrt{\sin\theta}} = \pi$ **(07 Marks)**

Module-4

7 a. Solve $\left[y\left(x+\frac{1}{x}\right)+\cos y\right]dx + \left[x + \log x - x\sin y\right]dy$ **(06 Marks)**

b. A body in air at 25°C cools from 100°C to 75°C in 1 minute, find the temperature of the body at the end of 3 minutes. **(07 Marks)**

c. Prove that the system of confocal and coaxial parabolas $y^2 = 4a(x + a)$ is self orthogonal. **(07 Marks)**

OR

8 a. Solve: $xyp^2 - (x^2 + y^2)p + xy = 0$ **(06 Marks)**

b. Solve: $\frac{dy}{dx} + y\tan x = y^3 \sec x$ **(07 Marks)**

c. Solve the equation $L\frac{di}{dt} + Ri = E_0 \sin wt$ where L, R and E_0 are constants and discuss the case when t increases indefinitely. **(07 Marks)**

Module-5

9 a. Find the rank of the matrix $A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix}$ using elementary row operation. **(06 Marks)**

b. Find largest eigen value and eigen vector of the matrix $\begin{pmatrix} 4 & 1 & -1 \\ 2 & 3 & -1 \\ -2 & 1 & 5 \end{pmatrix}$ by taking $(1, 0, 0)^T$ as initial eigen vector by Rayleigh's power method (perform 6 iteration). **(07 Marks)**

c. Solve the system of equations $x + y + z = 9$; $x - 2y + 3z = 8$; $2x + y - z = 3$, by Gauss Jordan method. **(07 Marks)**

OR

10 a. For what value of λ and μ the system of equations $x + y + z = 6$; $x + 2y + 3z = 10$; $x + 2y + \lambda z = \mu$ has i) No solution ii) Unique solution iii) Infinite number of solution. **(06 Marks)**

b. Reduce the matrix $A = \begin{bmatrix} 4 & 3 \\ 2 & 9 \end{bmatrix}$ into the diagonal form. **(07 Marks)**

c. Solve the system of equations $83x + 11y - 4z = 95$, $7x + 52y + 13z = 104$, $3x + 8y + 29z = 71$ by Gauss Seidal method (carry out 4 iteration). **(07 Marks)**

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